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JUDGMENT OF LIGHTNESS AND DARKNESS: AN INVESTIGATION OF MAGNITUDE JUDGMENTS OF AN INVERSE ATTRIBUTE

by

© RONALD CURTIS LAYE

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "Judgment of Lightness and Darkness: An Investigation of Magnitude Judgments of an Inverse Attribute" submitted by Ronald Curtis Laye in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

In many psychophysical studies subjects are called upon to make quantitative judgments of members of a set of test stimuli varying in magnitude of a physical property. The relation between physical value ϕ and judgment *J* is given by a power function, $J = a\phi^n$, where *a* is a fitting constant and *n* is the exponent which is stable for any particular continuum. There is often curvature in the plot of *J* against ϕ^n, especially for low values of the stimulus. The function may be made more linear by including an additive constant on either the stimulus axis or the response axis.

Sometimes subjects are instructed to judge the inverse attribute, such as softness of sound or darkness of gray, rather than the primary attribute of loudness or lightness. The power function provides good fits to such data, and the exponent has the same absolute value as for the primary attribute, but it is negative in sign (-*n*).

In the present study 32 subjects judged 8 gray stimuli presented in random orders 5 times each, and then judged differences between members of all 28 possible pairs formed from the same 8 stimuli 3 times each. All judgments were magnitude estimations, with half the subjects judging the attribute lightness, and half judging darkness.

In order to decide upon the most appropriate form of the power function, the absolute values of the exponents for lightness ($\underline{n}$) and darkness ($-\underline{n}$), were compared for both the stimulus constant and response constant forms of the power function. A fairly good match was obtained with the response constant, while the values for the stimulus constant differed considerably. On this basis the response translated power function was preferred.

The major purpose of the current research was to account for the negative exponent obtained for judgments of darkness by testing two alternative hypotheses. The two-stage model, which proposes separate power functions and, therefore, exponents, for the input process (relating ϕ to subjective value) and the output process (relating subjective value to $\underline{J}$) was applied to the difference judgments in order to discriminate between the two hypotheses. The attribute reversal hypothesis holds that the negative continuum exponent ($-\underline{n}$) for darkness is due to a reciprocal relation in the input process leading to a negative input exponent. Alternatively, the scale reversal hypothesis holds that subjective values for lightness are the same, and that there is a reciprocal transformation in the output process resulting in a negative output exponent. The scale reversal hypothesis was demonstrated to hold for the data, and the attribute reversal hypothesis was invalidated.

The results were discussed in terms of their relation to the literature. The standard scale of Munsell values was recovered with virtually no error from the input relation of the two-stage model. The concept of balanced stimulus range was developed and discussed in terms of the data and was related to other published data.

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INTRODUCTION

The two primary purposes of the present research were: (1) to investigate three alternative expressions for the relation between estimated magnitude and physical intensity of stimuli, and (2) to analyze judgments of an inverse attribute (darkness, as opposed to lightness) in terms of a two-stage model for magnitude estimation. With respect to the former problem, the relation between judged magnitude and physical measures of the stimuli is frequently expressed as a power function of the form

$$J = a\phi^{n}, \qquad (1)$$

where ϕ is stimulus magnitude expressed in physical units, $\underline{J}$ is judged magnitude of the stimulus, and $\underline{a}$ is a fitting constant reflecting the units of measurement. The exponent $\underline{n}$ is thought to be dependent upon the particular continuum, but it may be affected by specifics of the experimental conditions. The subjects' judgments are often obtained by the method of magnitude estimation, in which subjects are instructed to assign numbers in direct proportion to subjective magnitudes of the stimuli. An early review of the power function and judgment procedures was presented by Stevens (1957).

Although a plot of judged magnitude against physical intensity on double logarithmic coordinates is usually close to linear (as implied by Equation 1), there is often

systematic curvature in the plot. This departure from linearity sometimes is corrected by introducing an additional constant to the power function expressed in Equation 1. The constant may be included as an additive translation of the stimulus values, expressed as

$$J = a(\phi - b)^n. \tag{2}$$

The additive constant, b, is often interpreted as the value of absolute threshhold. Alternatively, when the constant is included as a translation of the response values, expressed as

$$J = a\phi^n + b, \tag{3}$$

b may be interpreted as a constant response scale error.

With respect to the second purpose of the present research, judgments of darkness have been found to be reciprocally related to judgments of lightness (Torgerson, 1960), and, therefore, the exponent of the power function for darkness is negative. Attneave (1962) has proposed a two-stage model for magnitude judgments. In the input stage the subject evaluates the subjective magnitude of the stimulus, and in the output stage the subjective magnitude is mapped into the number continuum. Each stage is characterized by a power transformation. The question is whether the negative exponent for a reversed continuum, such as darkness, stems from a negative input exponent or from a negative output exponent. The former implies that the subjects are evaluating the quantitative relations in terms of the reversed attribute, and the latter implies an

inversion of the response scale. These two alternatives are referred to as attribute reversal and scale reversal, respectively.

Additive Constant

Although the main reason for employing an additive constant seems to be to obtain a more satisfactory fit of the power function to the data, there have been attempts to justify the constant on theoretical grounds and to test its form empirically. The basic debate, reviewed by Marks and Stevens (1968), is concerned with whether the constant should transform the stimulus values or the response values. The power function with transformed stimulus values is given in Equation 2. The additive constant, b, is thought to be related to the value of sensory threshhold for the continuum in question. Stevens (1959), while acknowledging the utility of the stimulus constant b in straightening out the power function near threshhold, chose to interpret the constant as the value of absolute threshhold for the particular continuum. Ekman (1959) raised the possibility that the stimulus constant might be interpreted as noise in the sensory process which causes subjects' judgments to be greater than zero even when the value of the stimulus being judged is zero. The fact that the value of the stimulus constant is occasionally calculated to be negative led Corso (1963) to state that the absolute threshhold and the zero point of the physical

scale are two distinct factors contributing to the value of the additive stimulus constant. The data of several studies (e.g., Mashhour & Hosman, 1968), in which the value of the stimulus constant has been demonstrated to depend upon the value of the standard stimulus employed, weigh against an absolute threshhold interpretation of the constant.

When the additive constant is considered to be a transformation of response rather than stimulus, the value of b (from Equation 3) is interpreted as a masking effect of intrinsic physiological noise (Fagot & Stewart, 1968; Lochner & Burger, 1961), or as a misplaced zero on the response scale without further theoretical interpretation (Curtis, Attneave, & Harrington, 1968). The former interpretation yields the expression $J = a(\phi^n - b^n)$, as pointed out by Marks and Stevens (1968), where b may be interpreted as absolute threshhold, and the latter interpretation yields Equation 3 directly. Both expressions are formally identical to Equation 3 for empirical purposes in that the transformation appears on the response axis. McGill (1960) appears to have originated use of a response constant, but Ekman (1959) introduced it in theory by stating that available information did not allow for empirical discrimination between it and the stimulus constant.

There have been several attempts to compare the stimulus constant and the response constant forms of the

power law. Fagot (1966) developed the theory and gave minimum data requirements necessary to estimate and compare parameters of the power functions for the stimulus, response, and simple forms of the law. Fagot and Stewart (1968) chose the response law over the stimulus law on the basis of the better fitting individual subject power functions for the brightness continuum. They stated that their evidence was overwhelming in favor of the response law, and conjectured that for their data, independent estimates of the threshhold parameter would also allow better fit for the response law than the stimulus law. Their study has been criticized (Marks & Stevens, 1968) for using a standard stimulus which might prejudice the results. It can also be noted that their standard of 10 foot-lamberts is of considerably more luminance than any of the test luminances (.005 - 5.00 foot-Lamberts) which were chosen near absolute threshhold in order to achieve maximum powers of discrimination between the two additive constant forms. Mashhour and Hosman (1968) reported that the physical value of the standard in relation to the other stimuli greatly affects not only the value of the exponent, but also the value of the stimulus constant employed in their study.

Irwin and Corballis (1968) examined functions fitted to both loudness and softness judgments for response and stimulus laws and found that the response law provided a nearly reciprocal relationship (log-log plots of loudness

and softness having same absolute slope) for loudness and softness, while the stimulus law yielded an exponent for softness approximately three times as great (in absolute value) as that for loudness. In their study subjects judged the loudness of seven stimuli between 8 and 48 dB SPL and the softness of seven stimuli between 68 and 108 dB SPL. The loudness function showed the expected downward concavity toward the low SPL stimuli; the softness function also exhibited a steepening at the higher SPL values, indicating either a "maximum threshhold" interpretation of the stimulus constant or a very large value of 87.2 dB. The former could not be fit and the latter is unreasonalbe in magnitude. When the response constant form of the function was fitted to the data. Transformations on the response scale of 14.06 for loudness and 22.33 for softness were necessary and were interpreted as misplaced origin on the response scale. A basis for criticism of the study is that the subjects did not judge the same stimuli for loudness and softness, and in fact the two stimulus sets were considerably different and each of very limited range. Stimulus range has been demonstrated to greatly influence the exponent of the power function (Poulton, 1967; Teghtsoonian, 1971). Therefore, it may not be appropriate to choose two very limited ranges for the judgments and then choose the preferable functional form on the basis of matched exponent values.

In the present study an attempt was made to replicate

the Irwin and Corballis (1968) study, but using lightness and darkness of gray stimuli and just one stimulus set for both lightness and darkness judgments. Agreement between the absolute values of the exponents for lightness and darkness and the fits of the functions to the data were the criteria for the selection of the most satisfactory expression among Equations 1, 2, and 3.

Two-stage Model

It was suggested by Attneave (1962) that the basic power law of Equation 1 be modified to allow for theoretical separation of the stimulus input and response output processes. It was suggested that the relation between subjective magnitude Ψ and physical magnitude Φ and the relation between objective number N and subjective magnitude Ψ are both power functions, which may be expressed as

$$\Psi = a\Phi^{k} \tag{4}$$

$$N = c\Psi^{m}. \tag{5}$$

Combining Equations 4 and 5 yields

$$N = a'\Phi^{km}, \tag{6}$$

the two-stage expression relating number to physical magnitude. In tests of the two-stage expression it has been assumed that magnitude judgments are not necessarily on a ratio scale, and an additive constant, b, reflecting a displacement of the origin on the response scale, has given

$$J = N + b \tag{7}$$

and, combining Equations 6 and 7,

$$J = a'\phi^{km} + b, \tag{8}$$

for the relation between judgment *J* and physical magnitude ϕ. The latter equation is much like Equation 3 in that there is an additive translation on the response axis.

Judgments of single stimuli do not provide for empirical separation of *k* and *m*, but judgments made on pairs (or greater numbers) of stimuli do allow for such separation. Curtis, Attneave, and Harrington (1968) obtained judgments of differences between stimuli presented in pairs. The appropriate function derived from the two-stage model was

$$J_{ij} = a(\phi_j^{\,k} - \phi_i^{\,k})^m + b, \tag{9}$$

where ϕ_i and ϕ_j are members of the same stimulus pair and J_{ij} is the judgment for pair i,j. An assumption is that stimuli are input independently, and this allows for separation of *k* and *m*, whose product can be compared to *n* from the single stimulus data. This comparison has consistently verified the two-stage model for both group and individual subject data for several continua (summarized in Curtis & Rule, 1972b). Rule and Curtis (1973) have shown that the output exponent is the reciprocal of the power function exponent for subjective number, and, therefore, that the numerical responses given during magnitude estimation are not proportional to subjective values, but a power function of them, as required by the two-stage model.

In order to test the assumptions that the input and output transformations are actually power functions, Rule, Curtis, and Markley (1970) subjected geometric means of judgments of differences for circle areas and for weight to Kruskal's (1964) multidimensional scaling analysis from nonmetric information. Scale values, $\underline{V}$, were recovered based only on the assumption of monotonicity between the differences in $\underline{V}$'s and the judged differences. Power functions were found to provide good fits of scale values to physical values for both continua, verifying the power function transformation for the input stage. Next the difference judgments were fitted to differences between scale values, and again power functions provided good fits, indicating the appropriateness of power functions to express the output stage. The input and output functions combine to yield an expression equivalent to Equation 9, and all four parameters achieved through this original nonmetric solution matched closely the parameters obtained from a direct solution to Equation 9.

Curtis and Rule (1972a) have also solved directly by least squares for scale values by assuming the power function for the output stage. No assumption about the form of the input function was necessary at that point. The expression for the output function was

$$J_{ij} = (V_j - V_i)^m + b. \tag{10}$$

The $\underline{V}$'s are values on a scale whose origin, direction, and

unit are arbitrary. The parameters of the input stage were obtained by eventually assuming a power function of the form

$$V = a\phi^k + d, \tag{11}$$

where $\underline{d}$ is a fitting constant required because of the arbitrary origin of the $\underline{V}$ scale. Good correspondence between values of $\underline{k}$ achieved through the scaling solution and by direct solution to Equation 9 was obtained, but the scaling solution's values for $\underline{m}$ were somewhat low. Thus the assumption usually made, for direct solutions to Equation 9, about the form of the input function appears to be valid.

Judgment of Inverse Attribute

There is some debate concerning the observed differences in judgments made of an inverse attribute from those made of a primary attribute. Power functions usually provide good fits to data of both types, and the absolute values of the exponents from related primary and inverse attributes are approximately equal. For example, Torgerson (1960) found that the log-log plots of darkness and lightness of grays against physical magnitude had approximately the same slope (absolute value) except that the darkness slope was negative, indicating exponents approximately equal in absolute value and opposite in sign. Log-log plots of redness and paleness (Panek & Stevens, 1966) revealed a similar inverse relation, and there was

greater variability and curvature in the paleness (inverse) plot.

For judged darkness, JD, the power function expression equivalent to Equation for lightness is

$$JD = a\phi^{-n}, \tag{12}$$

Rather than pre-empt the answer to the question asked in this study about the location of an additive constant, the development in this section omits the constant entirely. Appropriate forms of these expressions may easily be derived including stimulus or response constants.

Since the continuum exponent *n* is equal to the product of *k* and *m* from the two-stage expression, a negative *n* must correspond to either a negative *k* or a negative *m*. A negative input exponent implies that subjective darkness, ψD, is inversely proportional to subjective lightness, ψL, thus yielding the input expression

$$\psi D = a\phi^{-k}, \tag{13}$$

and therefore,

$$JD = a\phi^{(-k)m} = a\phi^{-n}. \tag{14}$$

Torgerson (1961) indicated that the relation between magnitude estimates of darkness and lightness was due to a reciprocal relation between subjective darkness and subjective lightness. This situation may be referred to as attribute reversal.

Alternatively, if *m* is negative and *k* is positive, then the relation

$$JD = a\phi^{k(-m)} = a\phi^{-n} \tag{15}$$

holds. This implies that subjective darkness equals subjective lightness, and that the reciprocal transformation takes place in the function mapping subjective magnitude into numerical judgments, with number for darkness being inversely proportional to number for lightness. This alternative may be referred to as scale reversal, and it is similar to a hypothesis put forward by Stevens and Harris (1962) to explain judgments of roughness and smoothness of emery papers.

In the present study, the two alternative hypotheses expressed in Equations 14 and 15 were tested. In addition to single stimulus magnitude judgments, magnitude judgments of differences in paired stimuli were obtained for lightness and darkness groups. Scale values were obtained from the fit of Equation 10 to the data. The attribute reversal hypothesis was tested by examining the relation (Equation 11) between darkness scale values and physical magnitudes for a negative input exponent. The scale reversal hypothesis was tested by examining the relation between darkness scale values and judged darkness raised to the 1/-m power for linearity.

Gray Continuum

Before turning to the research reported in the present study, it is important to consider some of the previous research on judgments of the gray continuum. Poulton and

Simmonds (1965) found that the order of stimulus presentation affects the value of the power function exponent. Reflectance of the background against which the stimuli are observed was shown to have a major effect on the judgments of the subjects (Warren & Poulton, 1960) and on the input exponent of the two-stage model (Curtis & Rule, 1972a). Egeth, Avant, and Bevan (1968) had subjects judge similarities and differences between pairs of grays presented against white (90%), gray (17%), and black (3%) backgrounds. They found that background reflectance influenced the shape of the scale such that differences between stimuli close in reflectance to the background were enhanced relative to stimuli far in reflectance from the background. The Munsell gray continuum has been described as being sensitive to experimental conditions and accompanied by a high degree of variability or noise. People in general are not familiar with the physical scale of measurement and they are not as experienced in estimating lightness and darkness of grays as they are in estimating length, heaviness, and numerousness.

Location of the standard in the set of stimuli was demonstrated to affect the two-stage output exponent $\underline{m}$ (Curtis & Rule, 1972a) and the exponent $\underline{n}$ of the single stimulus power function (Mashhour & Hosman, 1968). The latter researchers, however, have been criticized by Stevens (1969) for their use of an additive stimulus constant and for not taking into account regression effects

in the cross-modal matching task. Poulton, Simmonds, and Warren (1968) found that subjects tend to use numbers ending in zero and five more than other available responses, but in averaging over several subjects and trials, the effects of this finding would be quite minimal.

Exponents of the power function for judgments of gray stimuli have been reported to range from .41 to 1.20. Mashhour and Hosman (1968) used eleven stimuli having reflectances between .033 and .685 in their study. Three different standards produced exponents between .86 (low standard) and .46 (high standard). Stevens (1969) reanalyzed their data averaging across standards, and recovered an exponent of .77. Curtis and Rule (1972a) employed a stimulus range of .046 to .648 with both high and low standards and high and low reflectance background conditions. The background reflectances mostly affected the input exponent value, and the standard stimulus reflectances mostly affected the output exponent value. For the relevant high reflectance background condition, $\underline{k}$ was .34 and .39, depending on standard location, $\underline{m}$ was 1.20 and 1.74, and $\underline{n}$ was .41 and .55. The low reflectance background produced substantially reduced $\underline{k}$'s, which could be considerably inflated by subtracting the background reflectance from the stimulus values before fitting the power function.

METHOD

Subjects

Sixteen men and sixteen women, all naive with respect to psychophysical scaling experiments, served as paid volunteers for one session each. The sessions lasted approximately one hour each. About half the subjects were enrolled in at least one summer session course at the University of Alberta, and some of the remainder had been university students previously. Age of the subjects varied from 20 to about 45.

Two additional subjects began the series of practice judgments, but were found by both the experimenter and themselves to be unable to adequately understand the instructions. Therefore, these two subjects did not begin the actual test series, and they were disqualified.

Materials and Apparatus

The stimuli judged were eight gray papers having Munsell values varying in uniform 3/4 steps between 7.25 and 2.0. The corresponding reflectances were .468, .362, .272, .198, .137, .090, .055, and .031.

A large enclosed plywood box containing its own constant flourescent lighting source served as the viewing apparatus. The rear inside wall against which the stimuli were presented was painted in flat white latex and

contained a 2 1/2 inch diameter hole for stimulus presentation on both the right and left sides. The frame of a stereoscopic slide viewer was mounted through the front wall, opposite the stimulus wall, and a vertical partition extending from the front wall about half-way to the rear wall insured that both right and left sides could be seen through the viewer by only the eye on the same side of the apparatus. The lighting scurce was not visible to a subject looking through the viewer. It provided the rear wall with 205 foot-candles of luminance. The reflectance of the rear wall was .977.

Procedure

Each subject was led by the experimenter into a room containing the apparatus, and was seated on a stool and cushion arrangement until the subject reported that he was at a height comfortable for viewing through the apparatus. Subjects performed the magnitude estimation of single stimuli before estimating differences. The instructions for the former task were read to the subject once slowly. Subjects were randomly assigned to either the lightness or darkness group. The only difference between the instructions for the lightness group and the darkness group was the substitution of the words dark and darkness for light and lightness in the instructions. The instructions for magnitude estimaticn of single stimuli were as follows:

> You will perform this task while looking through the viewer. You will be

presented with a number of gray papers having various lightnesses, and your job is to assign a particular number to the lightness of each gray paper according to the following rule: If a gray paper is lighter than the previously seen paper, you estimate how many times lighter it is, and multiply that number by the number assigned to the previous gray paper. For example, if the previous paper is called 25, and the current paper you see appears to be twice as light, you would call it 50, which is twice as big as 25. If it appears five times as light, you would call it 125, or 5 times 25. If it appears 3 1/2 times as light, you might call it 87, approximately 3 1/2 times 25. If, on the other hand, the present gray is not as light as the one you have just previously judged, then you must decide what fraction its lightness is of the previous one, and multiply the number assigned to the previous one by that fraction. For example, if the previous gray was called 25, and the current gray appears only 1/3 as light, you might call it 8, which is about 1/3 of 25. If it is 1/10 as light, you might call it 2.5, or 1/10 of 25. If it is half as light, you might call it 12.5. You may use only numbers greater than zero including fractions and decimals.

Do you have any questions?

Remember, if one gray is lighter than the previous gray, then you give it a higher number, depending upon how many times lighter it is than the previous gray. If it is less light than the previous gray, then you assign a number to it which represents what fraction its lightness is of the lightness of the previous gray.

When you have assigned a number to the lightness of one gray paper, then it will be removed, and you will be presented with another. You judge the lightness of this gray, basing your judgment upon the immediately preceeding judgment that you made. You will be presented with a number of such gray papers, and you are to judge each one until instructed to stop. You may

> take as much time as you need for each judgment, but you are urged to work as rapidly as possible so that you do not forget your judgment of the preceeding gray. You may ask me a question at any time concerning the task.
>
> We will now begin a series of practice trials. With which of your eyes do you prefer to see the gray papers? (Experimenter waits for response, and complies.) Look into the viewer, and you should see a gray paper with the eye you prefer. Just to begin with, we will call the lightness of this gray 10, and you will base the number you assign to the lightness of the next gray upon this.

A short series of practice trials followed the instructions so that it could be determined that the subject was following the instructions and making a judgment based on ratio rather than difference or some other rule. When the practice trials were completed, the subject was informed that the test series was to begin, and was told that he may call the first stimulus in the test series any number with which he felt comfortable.

Five random orders of the eight stimuli were presented to each subject (each received a different series of orders), so that each subject performed a total of 40 magnitude estimations. When these judgments had been completed, there was a five-minute break before the instructions for the difference judgments were read.

The instructions for magnitude estimation of differences were as follows (remembering that dark and darkness are substituted for light and lightness for

subjects assigned to the darkness group):

> Again this task will be performed while you are looking through the viewer. This time, however, you will see a gray paper on each side when you look, and instead of judging the lightness of each gray, your task is to judge the difference between the lightnesses of the two grays, compared to the immediately preceeding judgment you have made. Again, rather than judging lightness, you will be judging difference in lightness between the two grays. As with the previous task, you base your judgment upon your judgment of the immediately preceeding pair, according to the following rule: If there is a greater difference in lightness for the present pair than there was for the previous pair, you estimate how many times greater the present lightress difference is, and multiply that number by the number you assigned to the previous difference. For expmple, if you assigned the number 25 to the difference in lightness for the preceeding pair, and the difference in lightness for the current pair appears twice as great, you would call it 50. If it appears 1 1/2 times as great, you might call it 37 or 38. If, on the other hand, the difference in lightness for the present pair is smaller than the difference for the previous pair, you estimate what fraction the present lightness difference is of the previous difference, and multiply that fraction by the number you assigned to the previous difference. For example, if you assigned the number 25 to the difference in lightness for the preceeding pair, and if the difference in lightness for the current pair looks only 1/2 as large, you would call it 12 or 13. If it appears 4/5 as great, you would call it 20, or 4/5 times 25. You must base your judgment only upon the preceeding judgment you made, and may use any numbers greater than zero, including fractions and decimals.
>
> Do you have any questions?
>
> Remember, if the difference in

> lightness for one pair of grays is bigger than for the previous pair, then you give that pair a higher number, depending upcn how many times greater the difference is than the previous difference. If the difference in lightness for the pair of grays is less than the difference for the previous pair of grays, then you assign to it a number which represents what fraction its lightness difference is of the previous pair's lightness difference.
>
> When you have made a judgment, then that pair will be removed, and you will be presented with another pair. You then judge the lightness difference of that pair as compared with the difference of the immediately preceeding pair, and continue in this fashion until instructed to stop. You may take as much time as you need, but are encouraged to wcrk rapidly so that you do not forget your previous judgment. You may ask questions at any time.
>
> We will now begin a series of practice trials. Look into the viewer, and you should see your first pair of gray papers. We will call the difference in lightness for this pair 10, and you will base the number you assign to the next difference upon this.

A series of practice trials which typically was a bit longer than the series used fcr the first task was needed so that it could be verified that both the experimenter and the subject were confident that the subject was following the instructions. Sometimes it was necessary during the practice trials tc warn the subject of his tendency to use much larger ratios in moving in one direction on the scale than in the opposite direction. When the practice trials were completed, the subject was informed that the test series was to begin, and was told that he may call the

first difference any number he felt comfortable with.

Three random orders of the 28 pairs were presented to each subject (each subject received a different series of orders), so that each subject performed a total of 84 magnitude estimations of differences. The position (right/left) of the stimulus having the larger reflectance was consistent for individual subjects, but was balanced within lightness and darkness groups.

RESULTS

The data analyzed were geometric means of judgments for each stimulus or stimulus pair calculated over subjects and presentations for both the lightness group and the darkness group. These geometric means are presented in Appendices III and IV. The lines in the figures presented in this study were fitted by eye, and are simply suggestive of a possible relationship. The text may call attention to curvature in a plot of points through which is drawn a straight line, but upon examination, the reader will be able to detect the curvature. All functions were fitted by an iterative nonlinear least squares procedure with deviations of data points from predicted values all weighted equally. Solutions were also obtained by weighting each squared deviation by a factor inversely proportional to the square of the magnitude of the associated data point, and the resulting parameters are presented in Appendix I. Differences in parameter values for the two weightings employed were slight, and conclusions drawn from the data would not be affected by choice of the alternative weighting.

Additive Constant

Each of the three forms of the power function expressed in Equations 1, 2, and 3, repeated here for convenience,

$$J = a\phi^n, \tag{1R}$$

$$J = a(\phi - b)^n, \tag{2R}$$

$$J = a\phi^n + b, \tag{3R}$$

was fitted to the relations between magnitude estimation and physical reflectance for lightness and darkness conditions. The resulting parameters are displayed in Table 1. The values for the exponent all fall within the previously mentioned broad range for lightness of gray that has been reported in the literature.

An examination of Figure 1, in which judgments are shown plotted against reflectances on double logarithmic coordinates, reveals a definite downward concavity in the darkness plot. This curvature is similar to that of Torgerson's Figure 3-3 (Torgerson, 1960, p.28) for darkness of gray and can be seen regularly in data from magnitude estimation of inverse continua (Panek & Stevens, 1966). Thus, the exponent of the power function for lightness was little affected by choosing among the three models presented because there was little curve straightening possible for an additive constant of either variety. The exponent for darkness varied considerably among the expressions, because there was more curvature to be rectified, and an additive constant of either variety and of sufficient magnitude alters the slope of the log-log plot of the data versus the values of the function.

The simple power law solution of Equation 1 provided

Table 1

Parameter values for solutions to the simple, stimulus constant, and response constant forms of the power function (Equations 1, 2, and 3).

Function	Equation	Lightness			Darkness		
		a	n	b	a	n	b
$J=a\phi^n$	1	98.83	.65		3.32	-.73	
$J=a(\phi-b)^n$	2	98.12	.64	.003	2.21	-1.07	-.03
$J=a\phi^n+b$	3	98.73	.64	-.60	8.71	-.50	-8.70

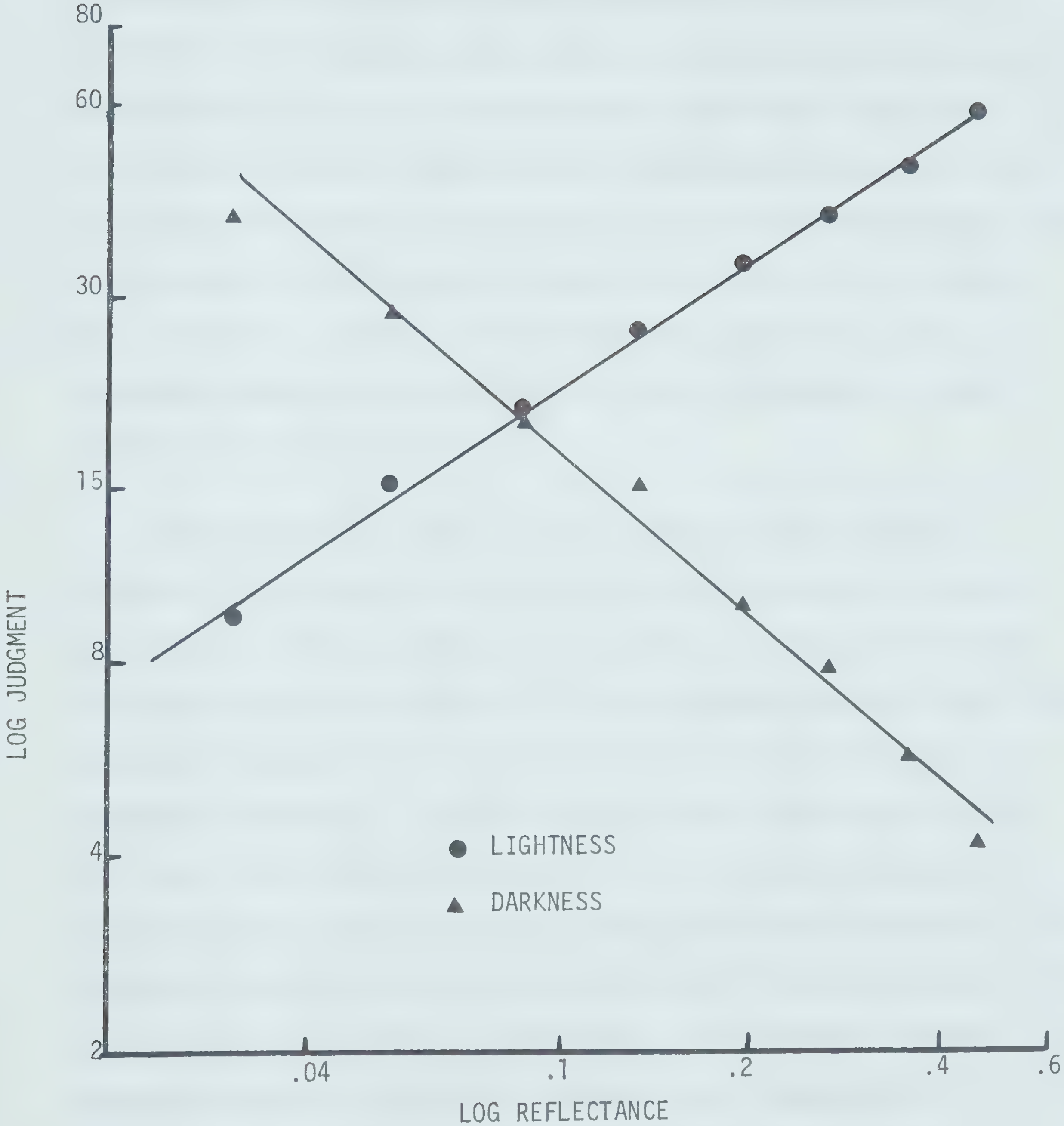

Figure 1. Log judgment versus log reflectance for lightness and darkness groups.

similar values of .65 and -.73 for lightness and darkness estimates of the continuum exponent. Note that the negative values of the darkness function exponent were due to the fact that the judgments are a decreasing function of physical reflectances, and that it is the absolute value of the exponent that is important for comparison with the lightness function exponent. The corresponding values of .64 and -.50 from the solution to the response constant law of Equation 3 also matched each other fairly well, while the values of .64 and -1.07 from the solutions to the stimulus constant form of the function (Equation 2) were considerably farther apart.

Judgments are shown as functions of the values predicted by each of the three forms of the power function in Figure 2. Examination of the plots reveals that a good fit is provided by all forms of the power law, but that the darkness function for the simple power law is subject to a downward curvature. It also appears that for each form of the power function, the lightness fit is better than the darkness fit. On the basis of these empirical findings, the response constant form of the power law (i.e., Equation 3) was chosen as the most appropriate. It provided similar estimates of the continuum exponent for lightness and darkness, and there was no apparent curvature in the relationships between the data and the values predicted by the functions.

The response constants, $\underline{k}$, obtained from solutions to Equation 3 for lightness and darkness were used to correct the subjects' judgments in Figure 4, which shows $\underline{J}$ (darkness) + 8.70 plotted against $\underline{J}$ (lightness) + .60. The fit can be compared favorably to the uncorrected fit shown in Figure 3. Note that if predicted values of the functions were substituted for the judgments in Figure 4, there would be no deviaticn from linearity. The points in Figure 4, then, are nearly colinear because the functions fit well, and it is the absence of systematic deveation that is more interesting than simple goodness of fit.

Another type of sclution was obtained for the single stimulus data. A single estimate of the exponent for lightness and darkness was obtained by simultaneously fitting each of the three forms of the power law to the lightness and darkness data. These results are displayed in Figure 5, showing the judgments as functions of the predicted values. The horizontal axes were labelled by the combined form of the functions, where (X1,X2) are dummy variables assuming the values of (1,0) and (0,1), respectively, depending upon whether the judgment is of lightness or of darkness. It can be seen that the response constant form of the power law provided the best fit, particularly for darkness and for the numerically smaller judgments. The other two forms of the power law exhibit some curvature in the darkness plots. The response constants -3.06 for lightness and -4.89 for darkness were

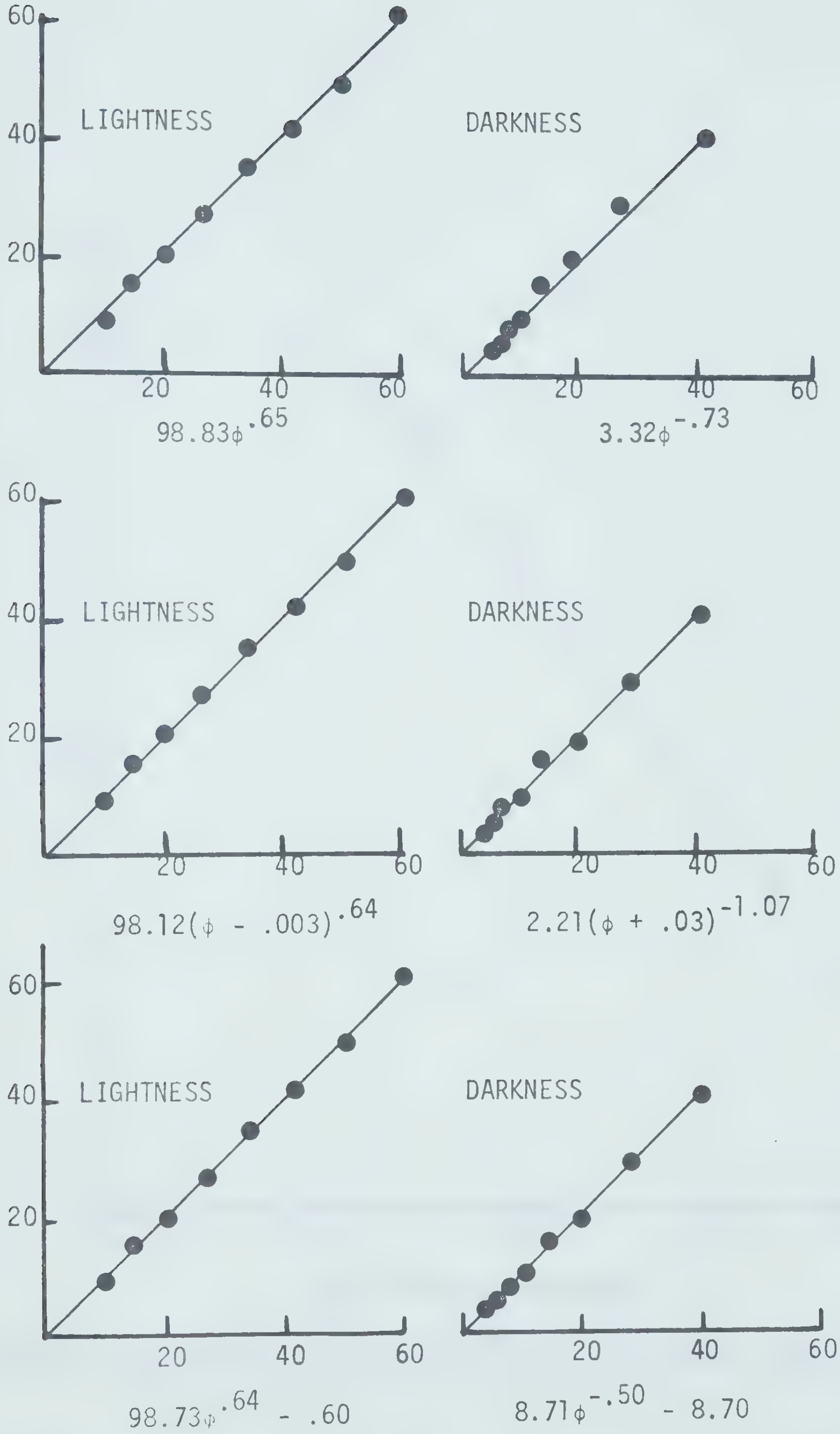

Figure 2. Judgment versus values predicted by three alternative forms of the power function for lightness and darkness.

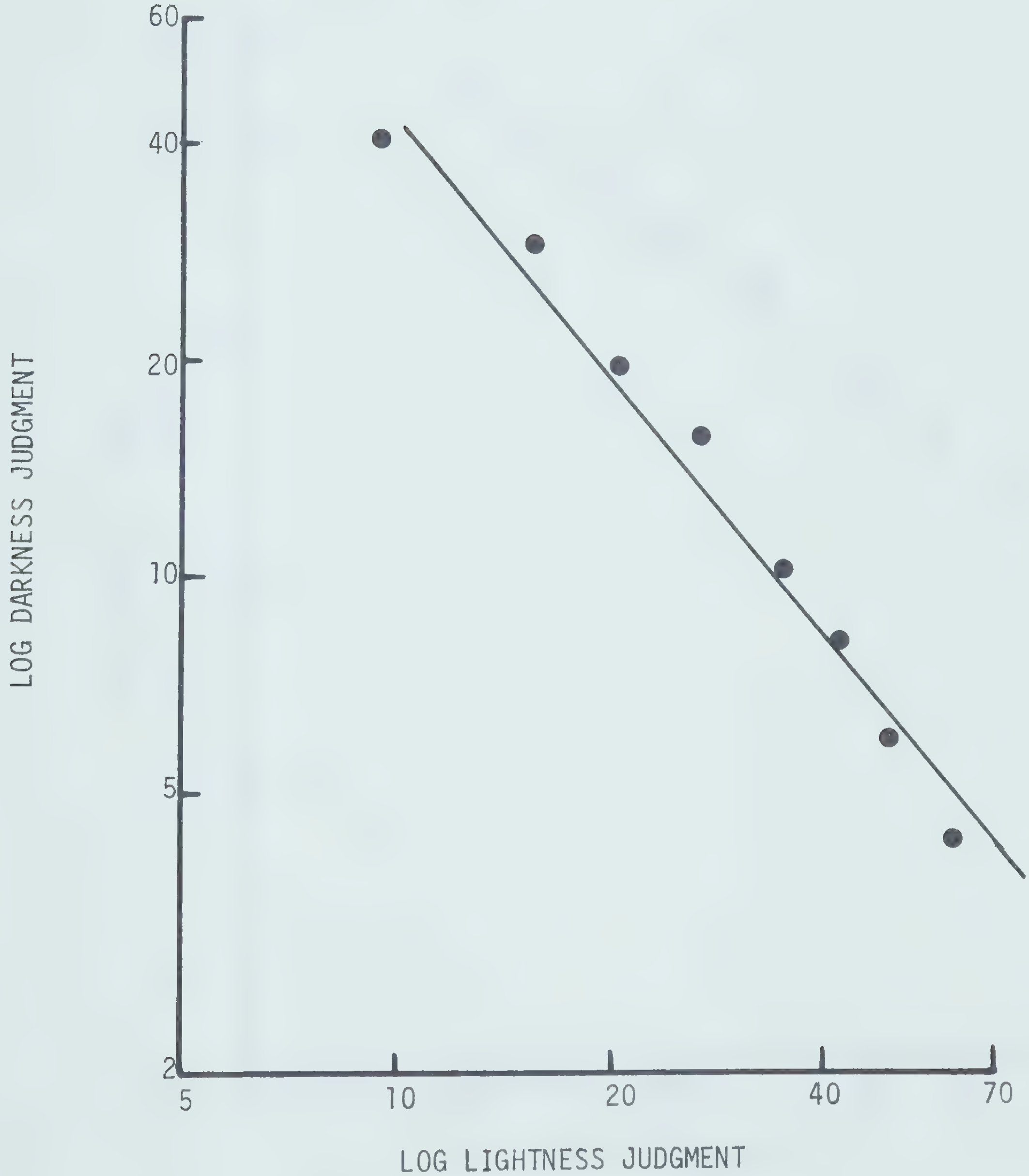

Figure 3. Log darkness judgments versus log lightness judgments.

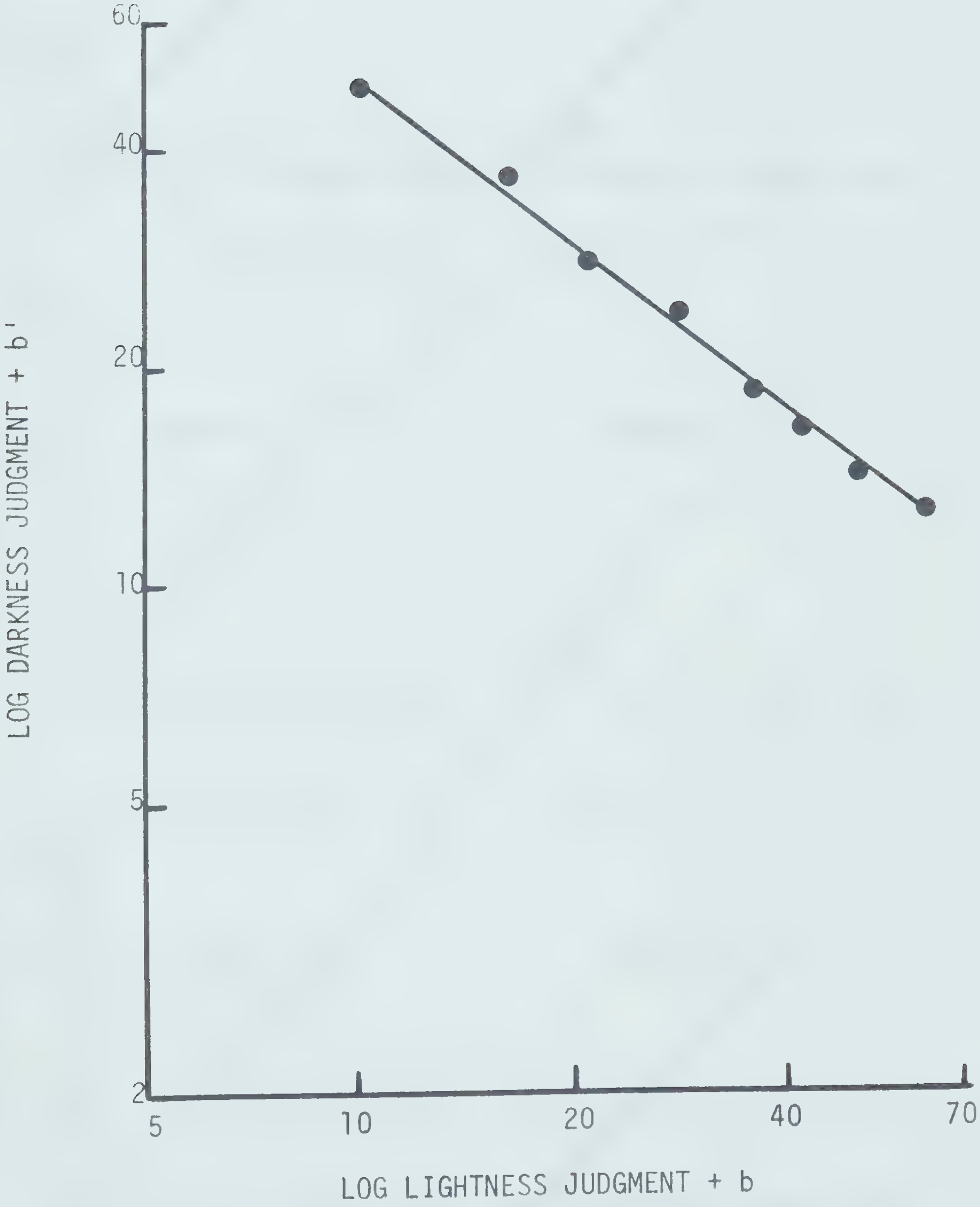

Figure 4. Log darkness judgments vs. log lightness judgments, with judgments corrected by additive response constants obtained from solution to Equation 3.

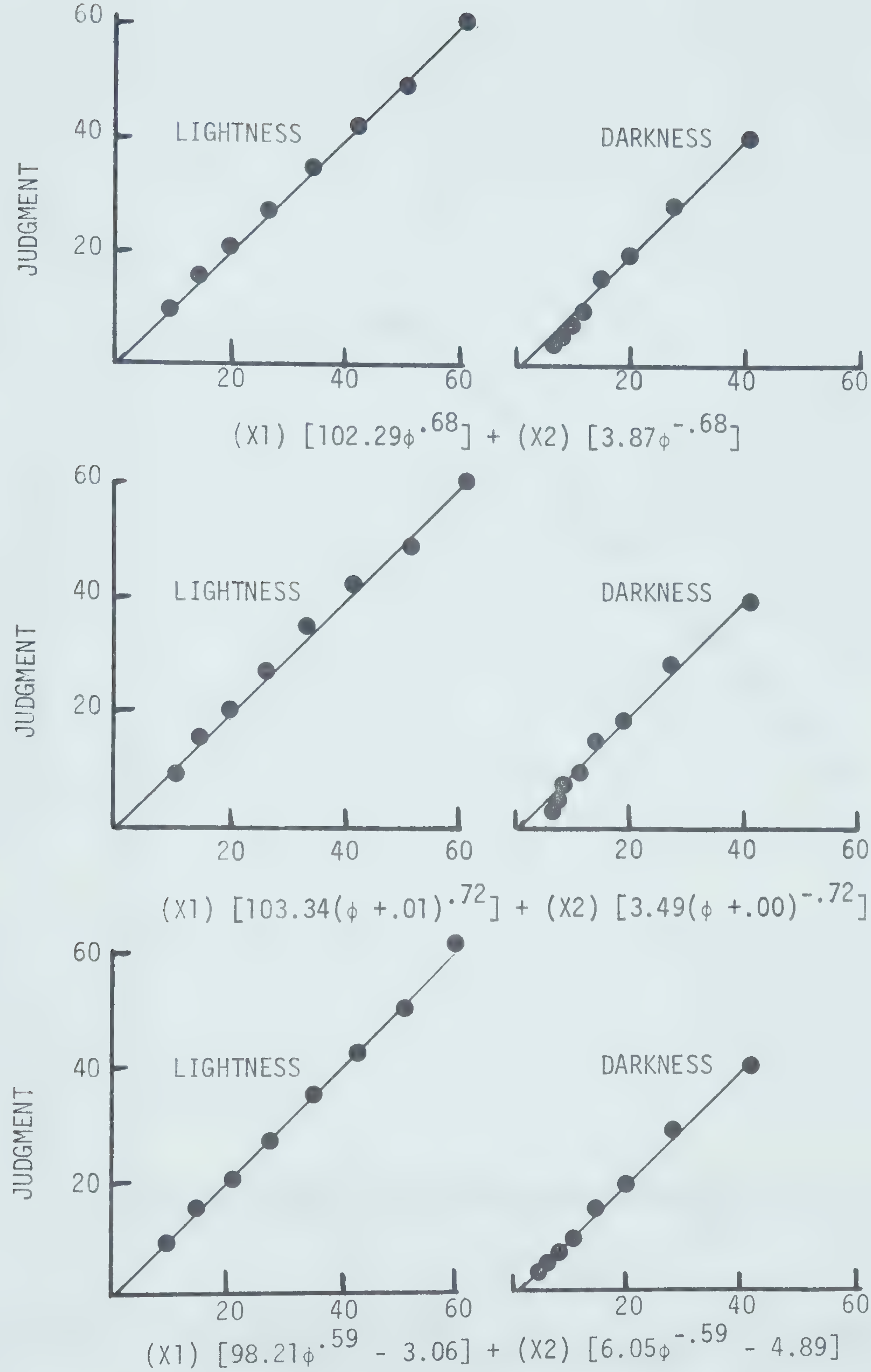

Figure 5. Judgment versus values predicted by three alternative simultaneous solutions for lightness and darkness.

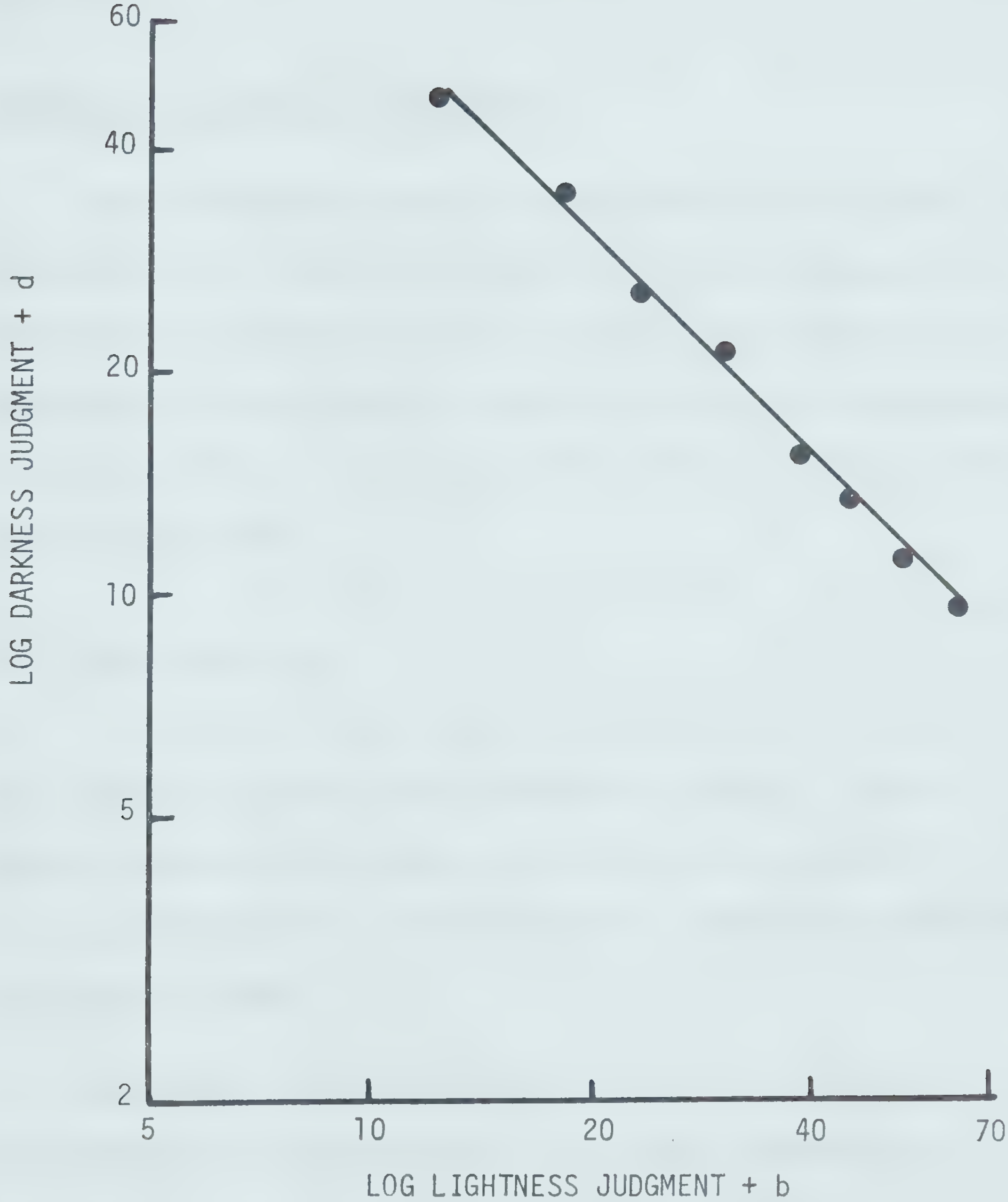

Figure 6. Log darkness judgment versus log lightness judgment, with judgments corrected by additive response constants from simultaneous solutions.

used to correct the *J*'s, and the resulting plot of Figure 6 for darkness versus lightness can be favorably compared to the uncorrected plot of Figure 3 and also with the corrected plot of Figure 4. These results have provided additional support for the response constant form of the power law.

Judgment of Inverse Attribute

The difference data for lightness and darkness were subjected to a direct analysis for scale values, *V*, developed by Curtis and Rule (1972a). The function of Equation 10 was fitted to the data with no assumption made about the form of the input expression. A least squares solution yielded

$$J = (V_j - V_i)^{2.04} + 3.39$$

for lightness, and

$$J = (V_i - V_j)^{1.78} + 5.52$$

for darkness. Difference judgments plotted against the values predicted by these solutions are presented in Figure 7; it appears that a power function adequately describes the output stage.

The scale values were fitted to the physical reflectances of the stimuli by a power function as expressed in Equation 11. The solutions were

$$V = 13.80\ \phi^{.30} - 1.01$$

for lightness, and

$$V = 21.02\ \phi^{.32} - 6.40$$

for darkness. Plots of scale value versus ϕ^k are presented in Figure 8. For each condition the power function provided a close approximation to the data. Scale values for darkness were demonstrated to be linear with scale values for lightness in Figure 9. Since the least squares solution for darkness, as well as for lightness, yielded a positive value for the input exponent, the data refute the attribute reversal hypothesis.

In order to test for a possible local least squares minimum residual for a negative value of $\underline{k}$ for the darkness input function, attempts were made to fit scale value for darkness to the expression $a\phi^k + d$ with $\underline{k} < 0$. The multiplicative and additive parameters, $\underline{a}$ and $\underline{d}$, were, as in previous solutions, not restricted as to sign or magnitude. The iterative procedure achieved no minimum, so there was no finite solution for negative $\underline{k}$. An examination of the residual error as a function of the value of the exponent indicated that there was no minimum for a negative input exponent. Predicted values were obtained, however, for the function $V = \phi^{-.32}$, and these were shown to be nonlinear with both predicted scale values obtained for darkness and for lightness in Figure 10.

In order to examine the adequacy of the scale reversal hypothesis for the data of the present study, the single stimulus darkness judgments were corrected by the additive constant $\underline{b}$ obtained from the solution to Equation 3, and

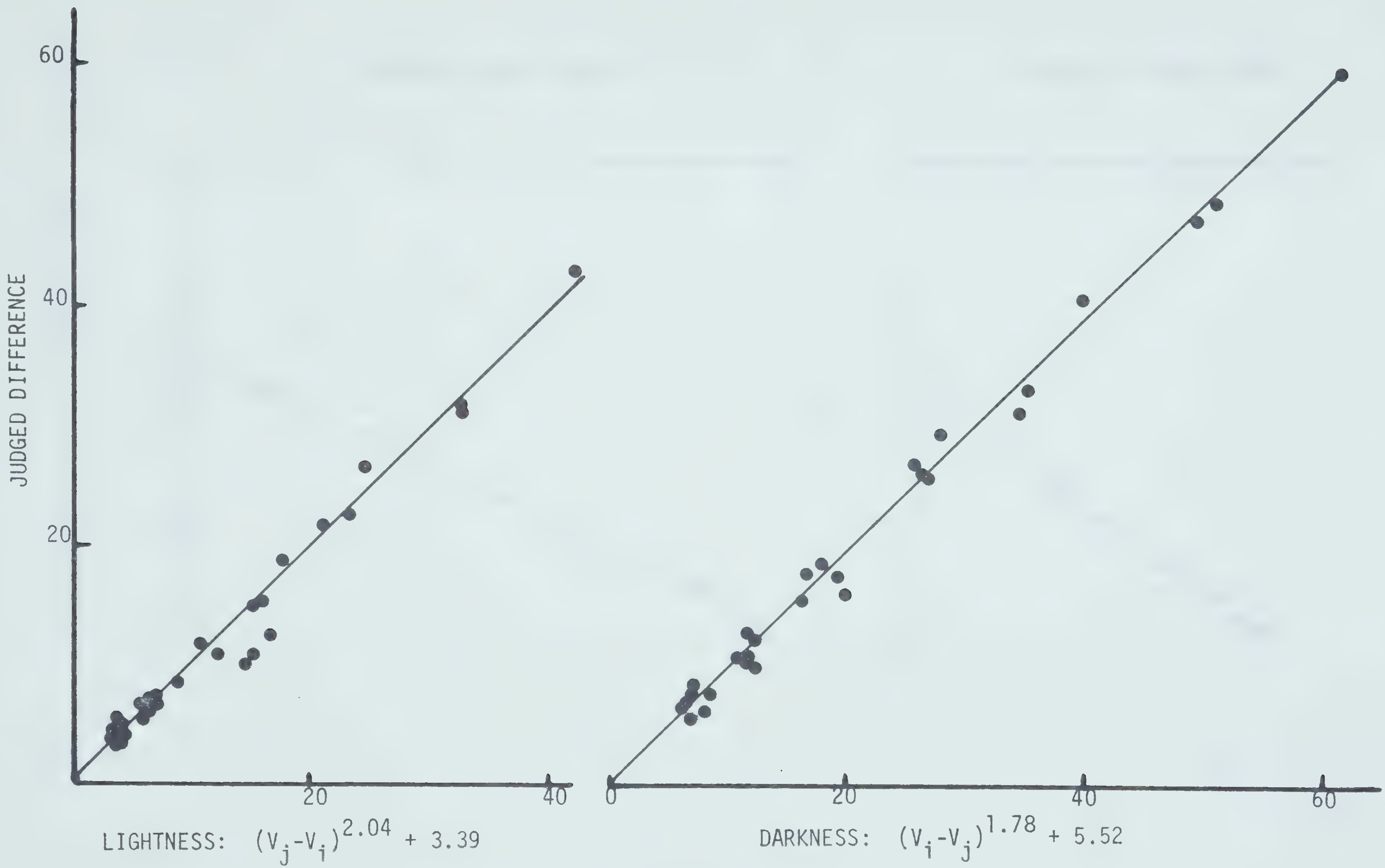

Figure 7. Judged difference versus value predicted by scaling solutions.

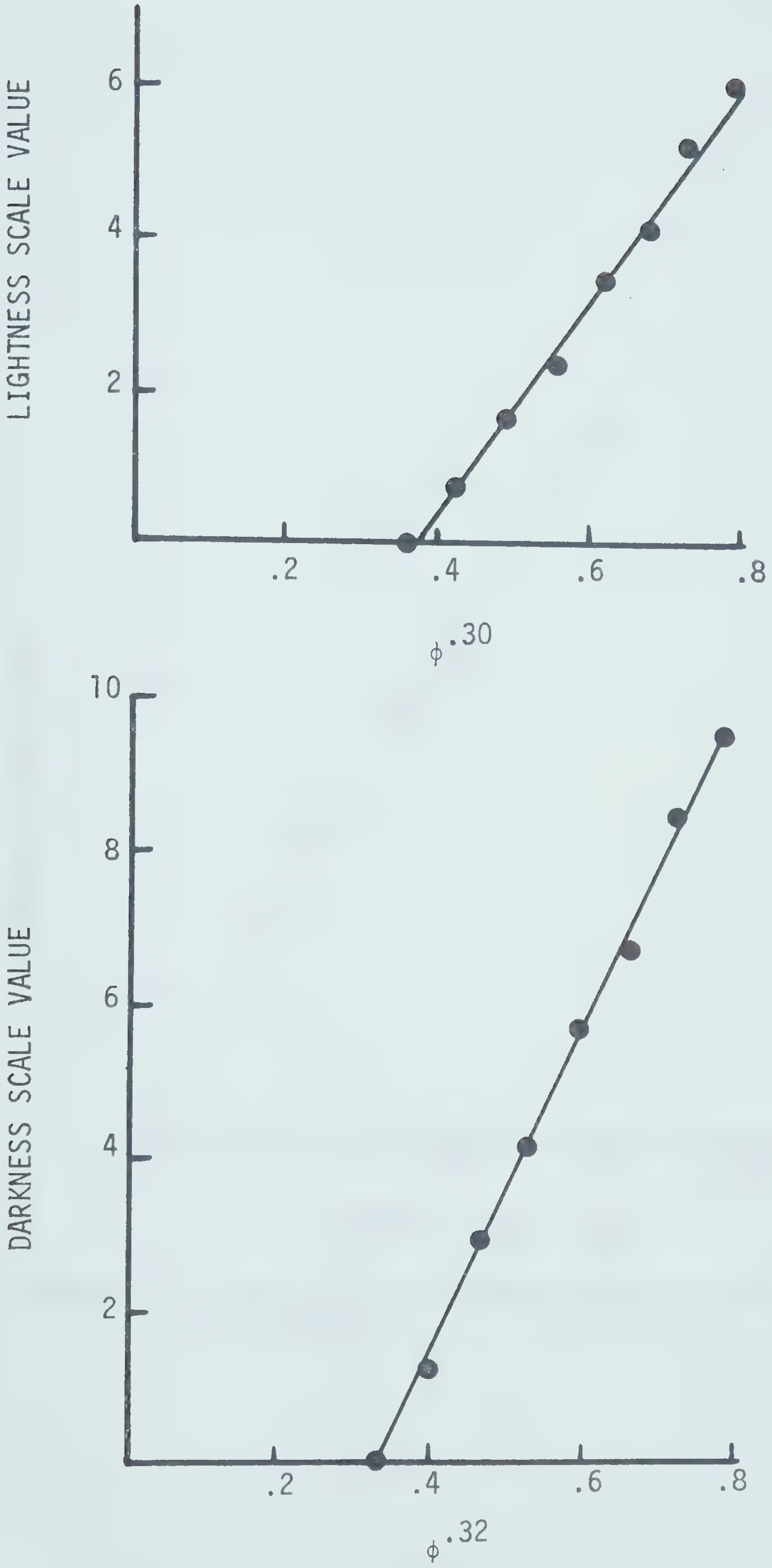

Figure 8. Scale value as a function of reflectance raised to the k power.

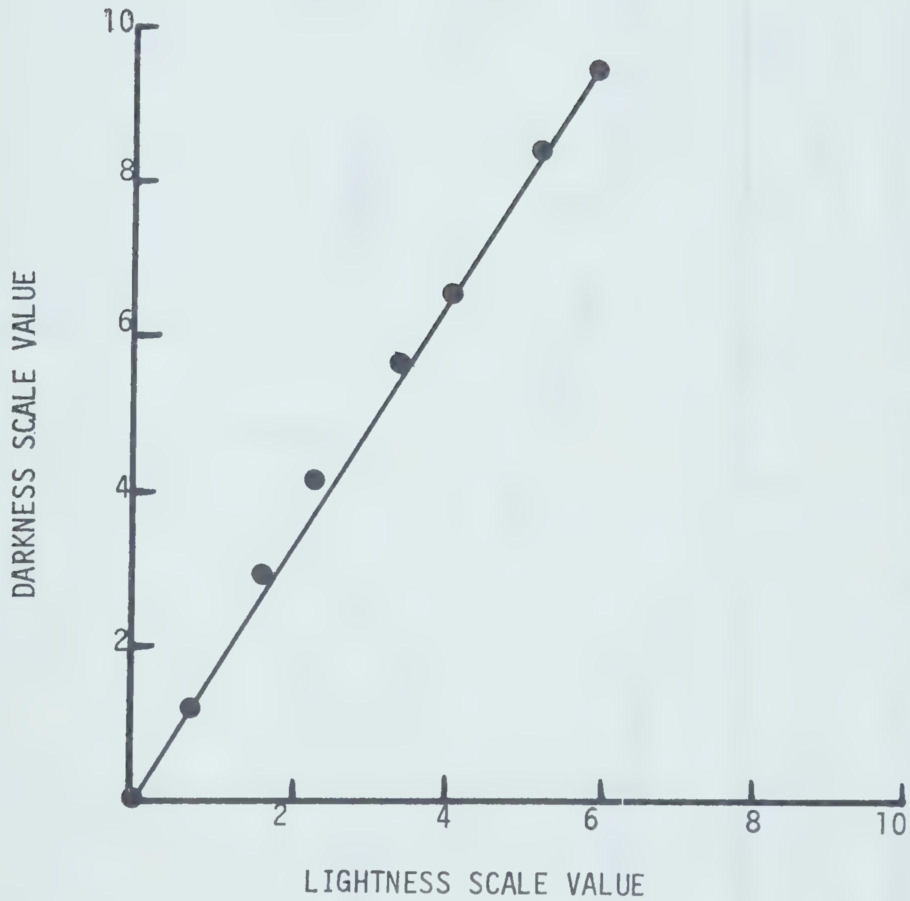

Figure 9. Scale value for darkness versus scale value for lightness.

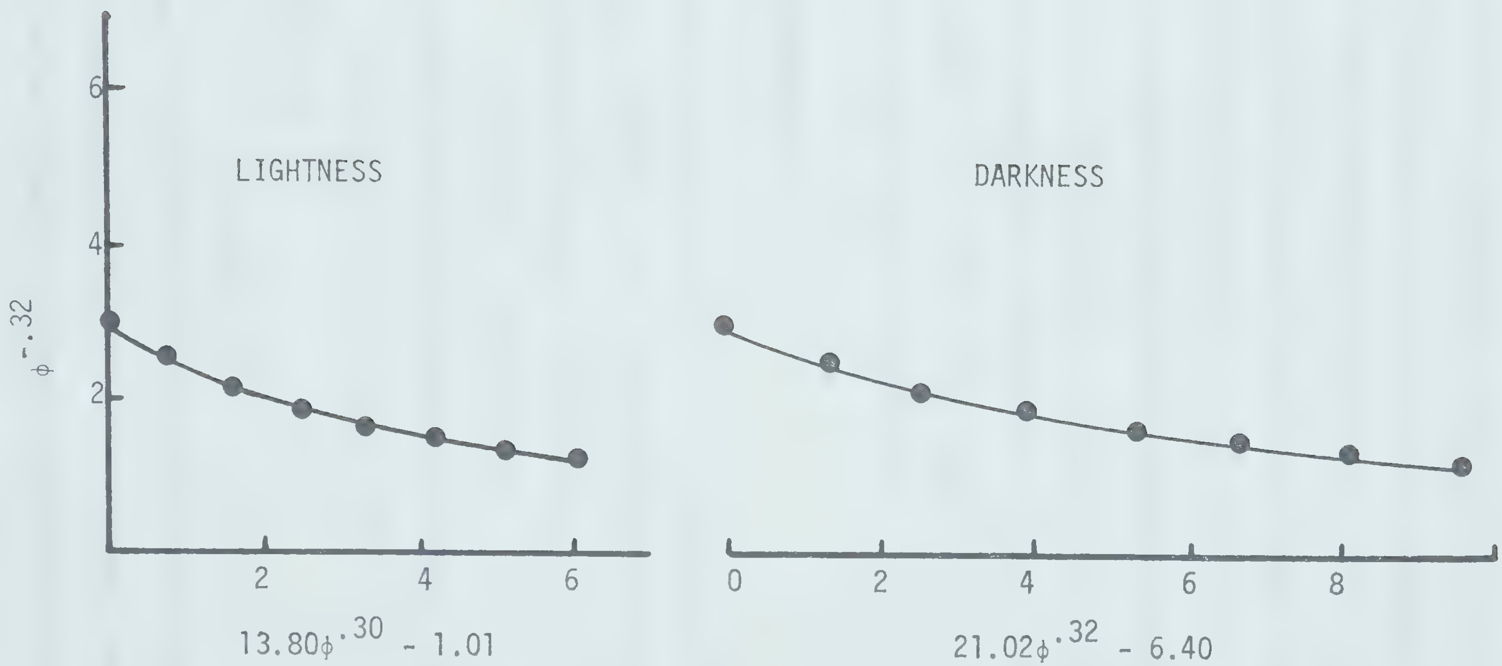

Figure 10. Reflectance raised to the -.32 power versus predicted values of lightness input function and of darkness input function.

the resulting values were raised to the 1/-m power, where $\underline{m}$ was obtained from the darkness solution to Equation 10. A linear fit of scale values to $(J - b)^{1/-m}$ is predicted from the scale reversal hypothesis. This would indicate a reciprocal transformation ($-\underline{m}$) for the output stage to account for the negative continuum exponent ($-\underline{n}$) from Equation 12 of the darkness function. Also, scale values for lightness were plotted as a function of $(J - b)^{1/m}$, where the parameters $\underline{b}$ and $\underline{m}$ were taken from corresponding solutions for lightness. These results for darkness and lightness are presented in Figure 11, and it can be seen from the linearity in both cases that the scale reversal hypothesis was supported by the data.

Two-stage Hypothesis

If a substitution of $(a\phi^k + d)$ from Equation 11 is made for $\underline{V}$ in Equation 10, allowing for a power function to represent the input stage, the function

$$J = a^m(\phi_j^k - \phi_i^k)^m + b \tag{16}$$

results. A substitution of parameter values yields

$$J = 213.94(\phi_j^{.30} - \phi_i^{.30})^{2.04} + 3.39$$

for lightness and

$$J = 226.90(\phi_j^{.32} - \phi_i^{.32})^{1.78} + 5.52$$

for darkness. The products of $\underline{k}$ and $\underline{m}$ from these solutions are .59 for lightness and .58 for darkness, which match each other quite closely and are reasonably close to the values of .64 and (absolute value) -.50 obtained from

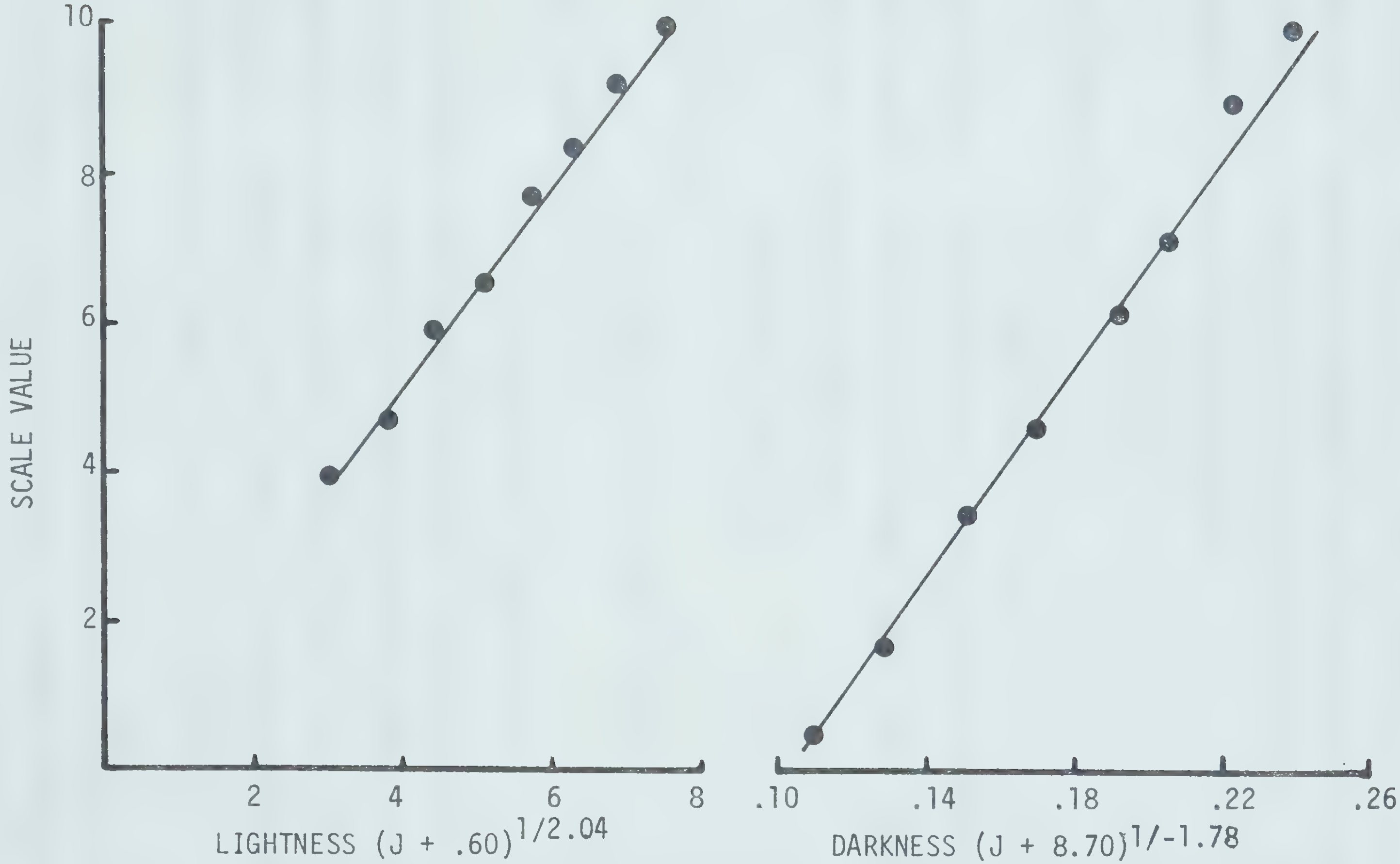

Figure 11. Scale value: a) for lightness versus judgment corrected for additive constant and nonlinear output exponent, and, b) for darkness versus judgment corrected for additive constant and nonlinear output function with negative exponent.

fitting Equation 3 to magnitude estimations of single stimuli.

An additional analysis of the difference data was performed by solving for the four parameter function of Equation 9 directly, assuming the adequacy of power functions for both the input and output stages. The following solutions were provided:

$$J = 196.58(\phi_j^{.30} - \phi_i^{.30})^{1.99} + 3.27$$

for lightness, and

$$J = 219.17(\phi_j^{.32} - \phi_i^{.32})^{1.75} + 5.35$$

for darkness. The fit of the data to the functions is described in Figure 12. Solutions were also obtained for the three parameter expression

$$J = a(\phi_j^{k} - \phi_i^{k})^{m}, \qquad (17)$$

and they were

$$J = 139.64(\phi_j^{.29} - \phi_i^{.29})^{1.52}$$

for lightness, and

$$J = 161.96(\phi_j^{.32} - \phi_i^{.32})^{1.31}$$

for darkness. Difference judgments were given as a function of values predicted by this expression with no additive constant in Figure 13. The plots reveal some systematic curvature similar to that found by Marks and Cain (1972) for judgments of weight, area, and roughness.

Table 2 lays out the comparisons of _k_ and _m_ products for the direct solutions with and without additive constant. The fact that the sclution to Equation 17, with

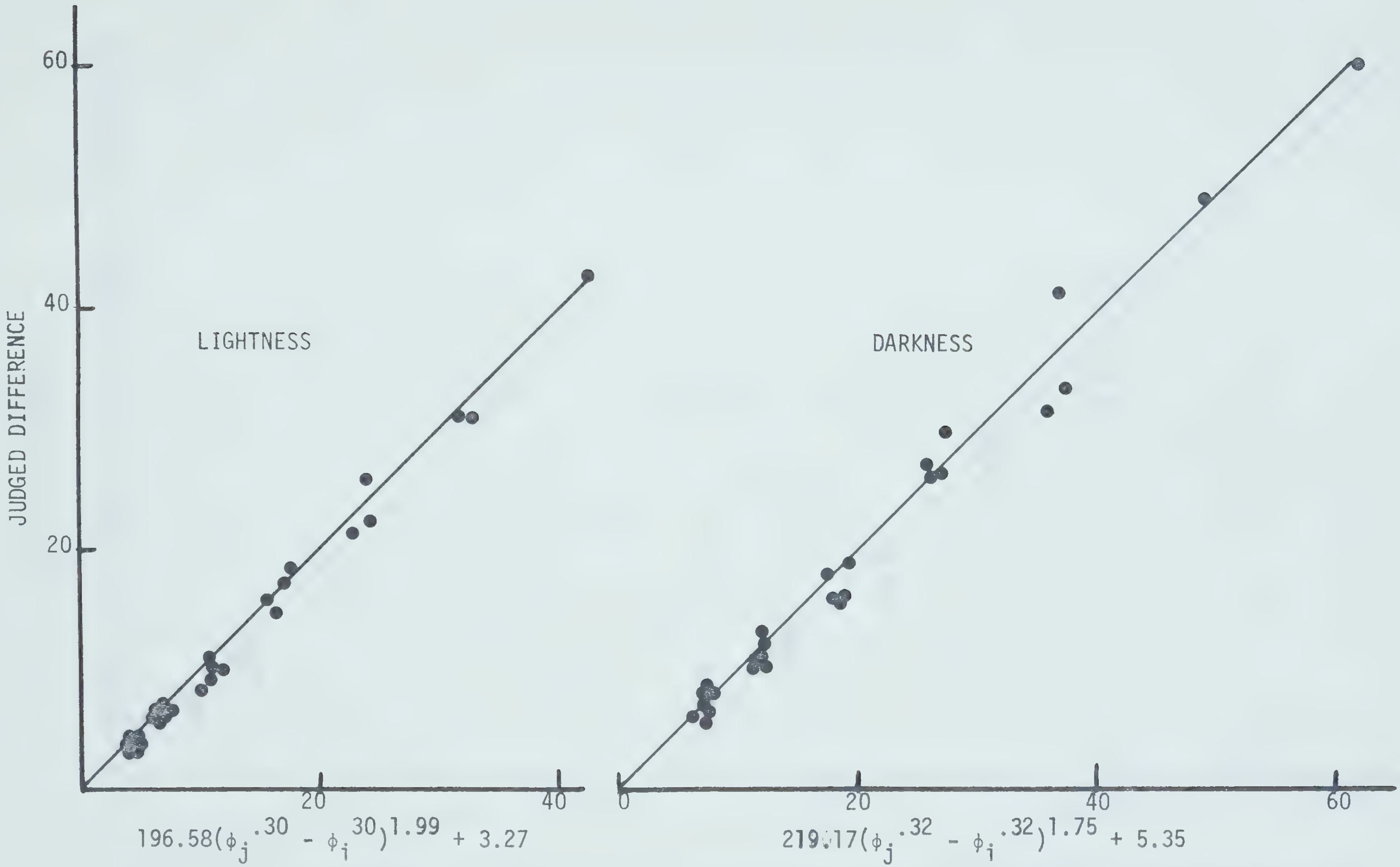

Figure 12. Judged difference versus values predicted by direct solutions to 4-parameter difference expressions.

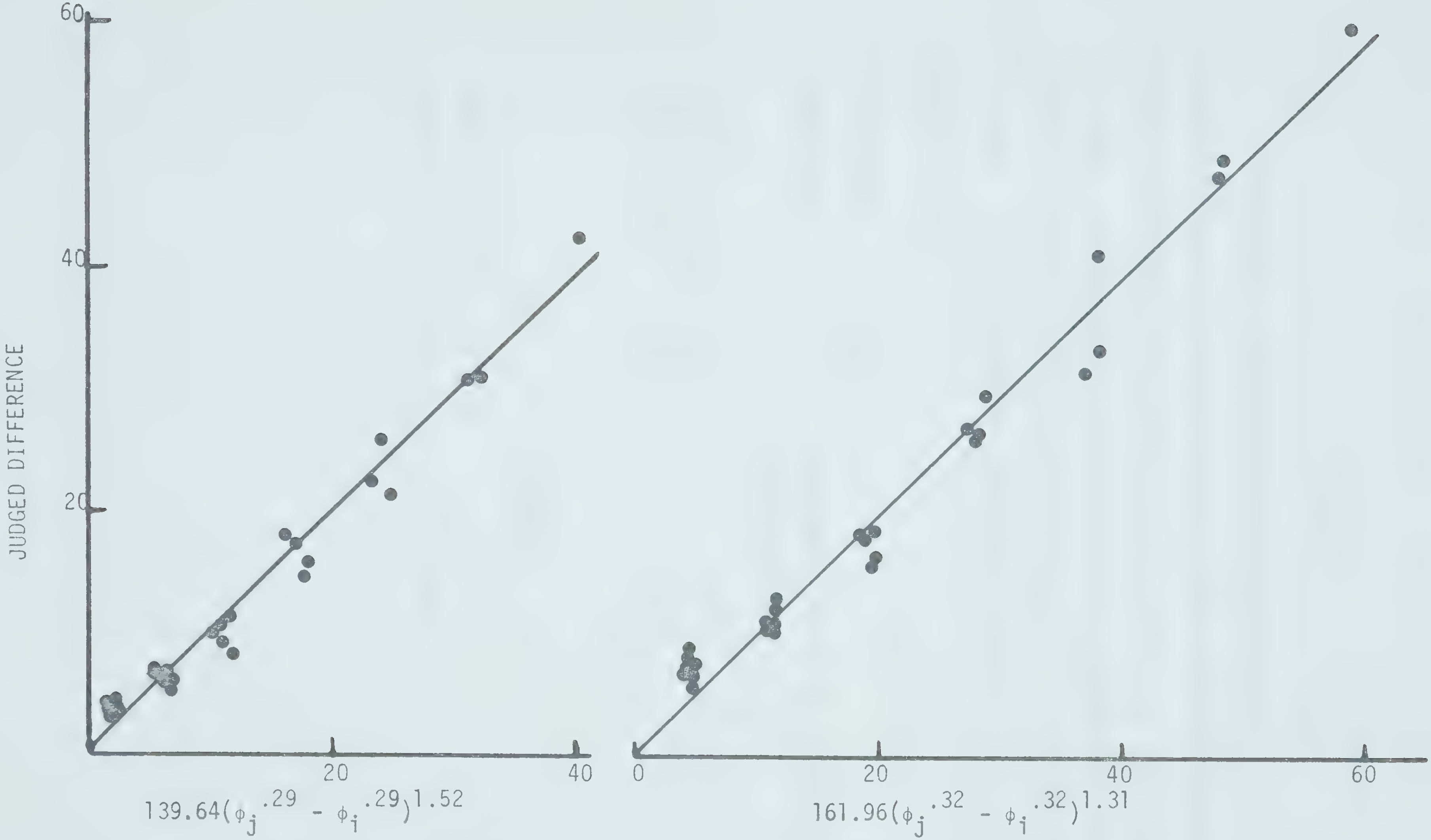

Figure 13. Judged difference versus values predicted by direct solutions to 3-parameter difference expressions.

Table 2

Comparison between the products of input and output exponents (*k*x*m*) from difference judgments fit to the two-stage expression and *n* from single stimulus judgments fit to power function and to simultaneous lightness and darkness solutions presented in Figure 5, for additive response constant and no additive constant forms of the expressions.

Two-stage Expression	kxm	n (Equation 1 or 3)	n (Simultaneous)
		LIGHTNESS	
$a(\phi_j^k-\phi_i^k)^m+b$	.60	.64	.59
$a(\phi_j^k-\phi_i^k)^m$	.44	.65	.68
		DARKNESS	
$a(\phi_j^k-\phi_i^k)^m+b$	.56	-.50	-.59
$a(\phi_j^k-\phi_i^k)^m$	.42	-.73	-.68

no additive constant, provided considerably poorer matches between the products and the exponents n from single stimulus solutions than did solutions with response constant provided additional support for the additive response constant form of the power law expressed in Equation 3.

Darkness judgments are shown in Figure 14 as a function of lightness judgments; a straight line describes the relation quite well, as was expected from the closeness of the k and m products for lightness and darkness for the solutions to Equations 9 and 16.

An additional attempt was made to discriminate between the scale reversal hypothesis and the attribute reversal hypothesis for the data in this study. It was already noted that Equations 9 and 11 provided a good fit to both lightness and darkness data, with the value of k for darkness similar to the value of k for lightness. An attempt was made to fit the expression $J = a(\phi_i^{-k} - \phi_j^{-k})^m + b$ for the darkness data. As with the scaling solution attempt to fit Equation 11 with k less than zero, no finite least squares solution could be obtained for this function, even when k and m were individually held constant at various values throughout their reasonable ranges in an attempt to iterate towards the value of the variable exponent parameter. Also, no fit could be obtained for the function $J \quad = a[(1 - \phi_i)^k - (1 - \phi_j)^k]^m + b$, where the

quantities $(1 - \phi)$ are measures of absorptance of the stimuli. Furthermore, no fit to the magnitude estimations of single stimuli could be found for the comparable expression $J = a(1 - \phi)^n + b$. Therefore, it appears that absorptance is not a useful physical measure of darkness for the present study in that it is not related by a power function to judgments of darkness. Torgerson (1960) indicated the possible use of such a complementary physical measure for continua bounded below and above.

The internal consistency of the data was tested by examining a set of transitivity relationships for all stimulus triads. First, all mean difference judgments were corrected for misplaced zero on the response axis and nonlinearity of the output function using parameters obtained from solutions to Equation 9. For each stimulus pair (i,j), the corrected judgment, J , is given by

$$\dot{J}_{ij} = (J_{ij} - b)^{1/m}. \qquad (18)$$

Then for any ordered triad of stimuli (g,i,j), the direct estimate of the exterior pair, $\dot{J}_{gj}$, may be estimated by $\dot{J}_{gi} + \dot{J}_{ij}$. These indirect estimates of exterior pairs were plotted against the direct estimates in Figures 15 and 16 for lightness and darkness. The plots show that the data satisfy the transitivity relation, thus providing additional support for the response constant form of the power law. Fagot and Stewart (1969) reported unfavorable results in their attempt to test the internal consistency of their data from judgments of differences, but they did

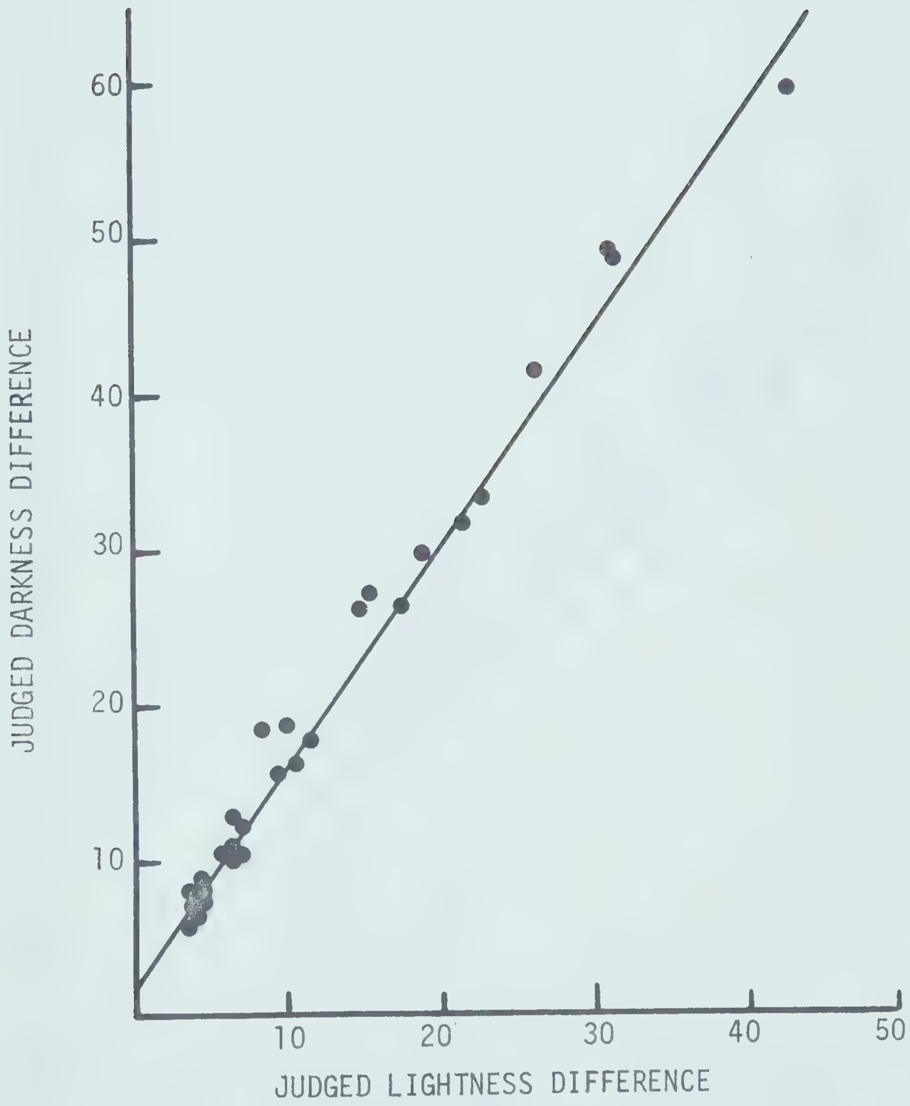

Figure 14. Judged darkness difference versus judged lightness difference.

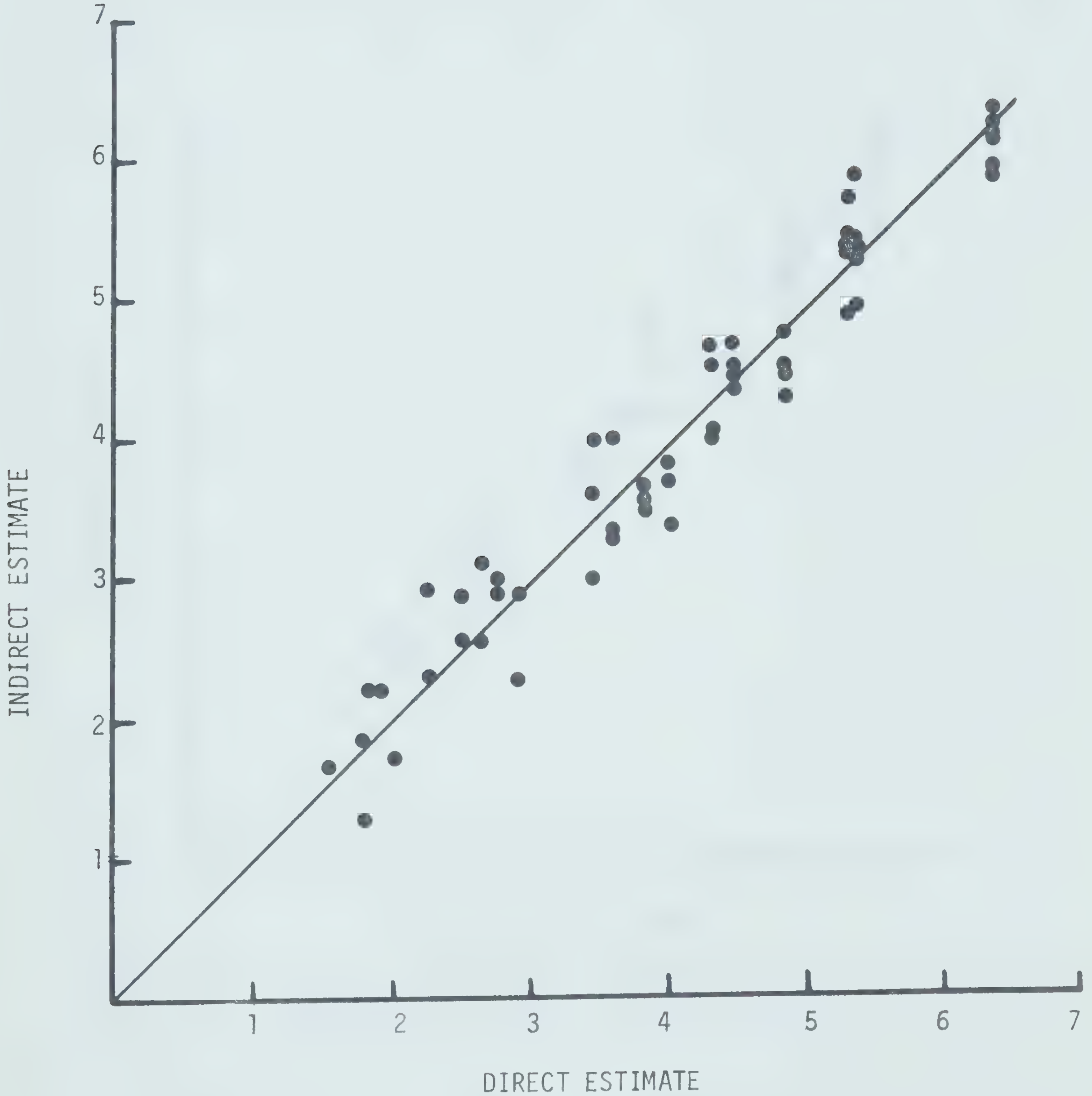

Figure 15. Indirect estimate of exterior pair versus direct estimate of exterior pair for lightness.

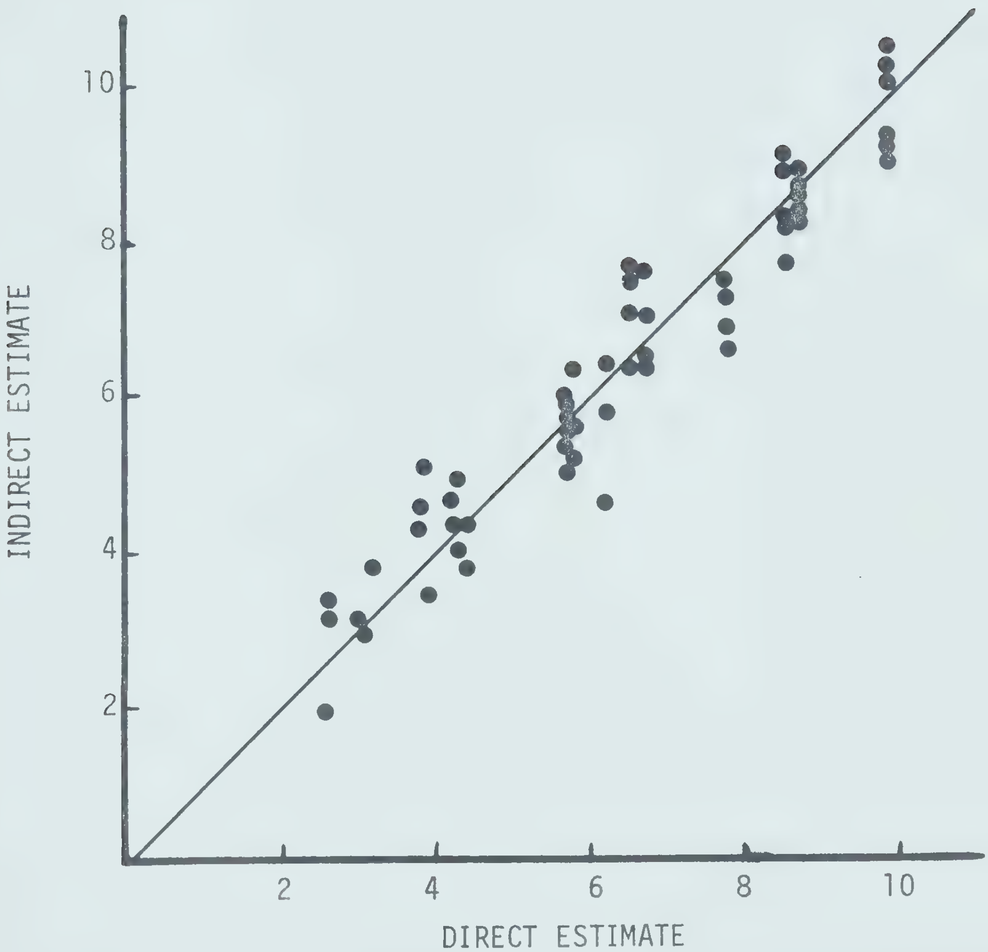

Figure 16. Indirect estimate of exterior pair versus direct estimate of exterior pair for darkness.

not include an additive constant in their correction expression. Results of the present study have shown that such a constant is a necessary part of the power function expression. The corrected values of exterior pairs, and their estimates by summed corrected interior pairs, and presented in Appendix II.

DISCUSSION

The primary evidence for including an additive constant as a translation on the response scale, as in Equation 3, was the agreement in the absolute values of the exponent *n* for lightness and darkness, coupled with the lack of systematic curvature in the plot of the judgments versus the values predicted by the function. When the constant was instead included as a translation on the stimulus axis, as in Equation 2, the estimates of the exponent differed greatly for lightness and darkness. When no constant was included in the power function expression, as in Equation 1, the plot of the judgments versus the predicted values of the function exhibited curvature.

Translation of the response scale was also preferred by Irwin and Corballis (1968) because it resulted in an approximately reciprocal relation between loudness and softness functions, while translation of the stimulus scale produced a softness function which was almost three times as steep as the loudness function. Also, Ross and Di Lollo (1972) found that change of stimulus range transformed the response scale by an additive constant, implying that such a constant might be interpreted in terms of the experimental conditions such as stimulus range.

Further evidence favoring the additive response constant was available from the analysis of transitivity

relationships for difference judgments. The response constants were necessary for a good linear fit of the interior pair sums to the exterior pairs. Furthermore, the products of $\underline{K}$ and $\underline{m}$ from the two-stage solutions with no additive constant did not closely match the values of the exponent $\underline{n}$ of the simple power function for either lightness or darkness, as they did when the response constant was included.

The data have been shown to support the hypothesis of scale reversal and refute the alternative of attribute reversal. A reciprocal transformation in the output function mapping subjective value into objective number accounts for the negative continuum exponent (of Equation 12) for darkness. Subjective values for lightness and darkness are the same, as shown by the agreement in the values for the input exponent for lightness and darkness ($\underline{k}$ = .30 and .32, respectively). Torgerson (1960) argued that the subjective scales for lightness and darkness are related by a reciprocal transformation, and that equal differences for darkness would not be so for lightness. Perhaps it was lack of discrimination between the subjective value and response (judgment) value which led to this invalid conclusion. His implicit assumption was that numbers were actually assigned in proportion to subjective value; that is, that the output exponent value was 1.0. This assumption has been demonstrated to be incorrect (Rule, 1969; Rule, 1971; Rule & Curtis, 1973), and the

value of the output exponent is found to be greater than unity.

It appears that the power relation between the subjective value and the physical value is robust; this may be an intrinsic property of the sensory transdeucers. However, the relation between the response and the subjective value is dependent upon the response method employed. Though fit by a power function in the two-stage expression, a linear output function may be appropriate for category methods of response (Curtis, 1970; Curtis & Fox, 1969). Torgerson (1960) suggested that a logarithmic relation exists between magnitude estimation and category rating data, but the curvautre in the relation is instead due to differences in the output function for the two procedures.

Stevens (1971) also implicitly assumed a linear output function for magnitude estimation. He presented a graph (Figure 4, p.432) for loudness of tones, depicting how judged magnitude of equal sone intervals decreases as a function of the physical value of the smaller stimulus defining the pair judged. The implication was that equal loudness intervals on the sone scale (i.e., a response scale) were subjectively equal and should, therefore, be judged equal regardless of the stimuli defining the intervals. Stevens proposed that subjects used a virtual exponent whose value was about half that of the continuum

exponent, and considered the interval estimation task to be afflicted by a systematic bias in this regard.

The results reported by Stevens can be explained most adequately within the framework of the two-stage model. Because the value of the input exponent for all continua thus far explored is less than the value of the continuum exponent, $\underline{n}$, intervals that are equal on a response scale constructed from magnitude estimation of single stimuli (such as Stevens' sone scale) will be unequal in subjective value, and the greater the physical magnitudes of the stimuli in the pair, the smaller the difference in subjective value. The data presented in Stevens' Figure 4, mentioned above, demonstrate the kind of nonlinearity predicted by the two-stage expression.

The data for Figure 17 were from the present study. Straight lines fitted through the points represent approximately equal intervals on the subjective scale for both lightness and darkness. The values of the input exponent were taken from solutions to Equation 9. There is no systematic curvature as the intervals move up the stimulus scale, and the lines are all nearly horizontal. Stevens implicitly defined the virtual exponent as the value required to produce such horizontal lines, and, therefore, the virtual exponent has the same value as the input exponent of the two-stage model. It is suggested here that the input exponent is the more theoretically developed

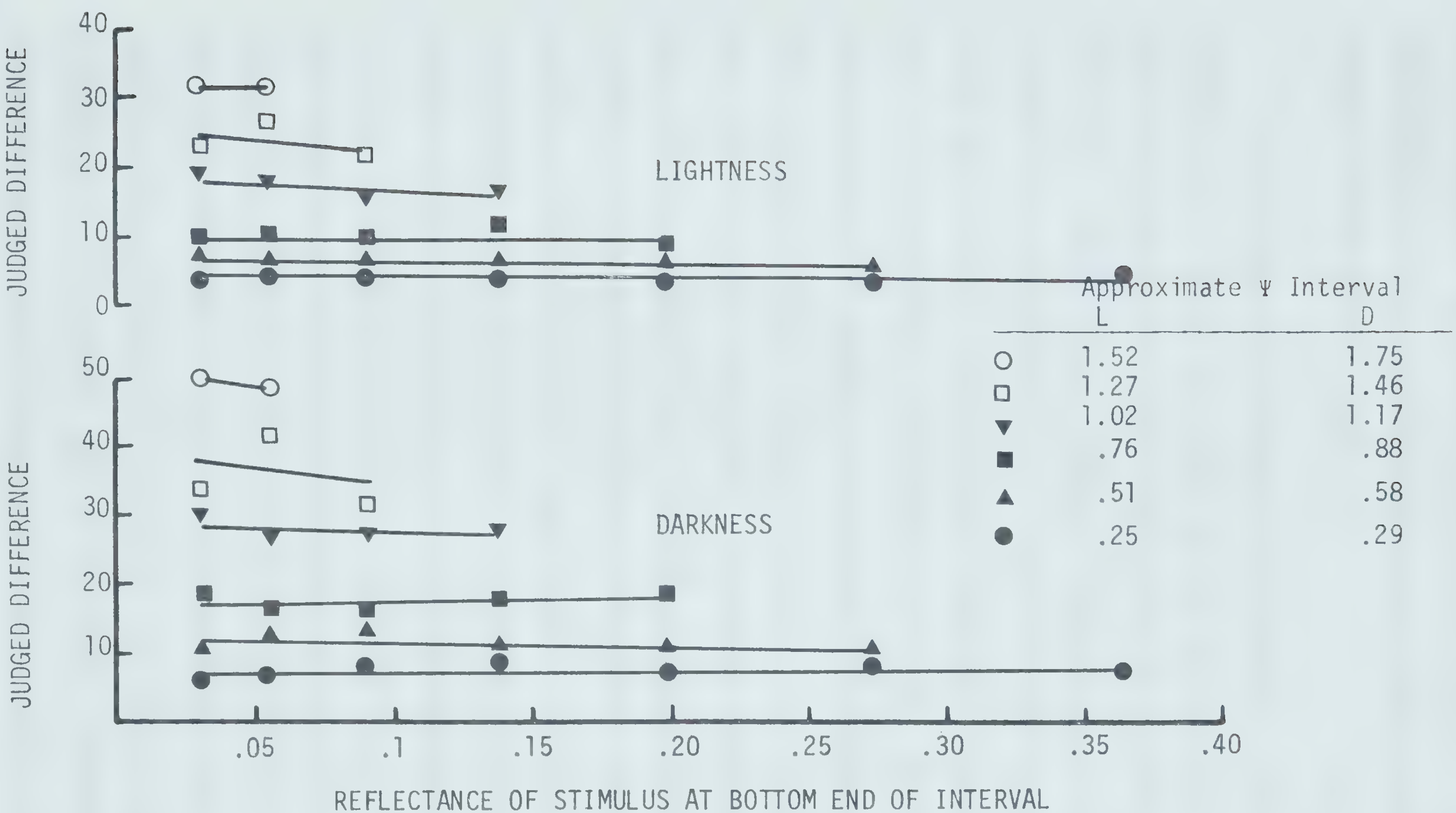

Figure 17. Judged difference for equal subjective intervals as a function of physical reflectance of stimulus at bottom end of interval. Straight lines are fit through points representing equal subjective intervals.

and integrated of the two concepts, and is, therefore, preferable to the virtual exponent, which appears to be without theoretical basis.

An approximate analysis of Krueger's (1970) data from judgment of length in terms of the two-stage expression revealed input and output exponents of approximately .78 and 1.30, respectively. These values would accurately predict the "apparent (i.e., judged) combined lengths" for pairs and quads of lines whose combined objective lengths were 1.5 inches and 3.0 inches, when the objective distribution of length was between 25%/75% and 40%/60%. (See Krueger, 1970, Tables 1, 2, and 3.) Krueger noted that both for the more unequally (5%/95% to 15%/85%) and for the more equally (45%/55% to 50%/50%) distributed objective lengths, the apparent (judged) combined lengths decreased somewhat, but were still considerably greater than the objective combined lengths. This possibly was due to the nature of the task, which may have encouraged subjects to do three different operations, depending upon the per cent distribution of objective lengths. In the first case, length was distributed moderately unequally, and it seems that subjects did as instructed; that is, they summed the lengths subjectively and reported the result. When the distribution of length was about equal, subjects may have recognized that and simply doubled apparent length, which would have them produce a line where the subjective scale value of the produced line would be equal to 2 raised to

the 1/m power times the subjective value of the judged lines. If $\underline{m}$ = 1.30 approximately, then $2^{1/m}$ = 1.71, and the ratio of 1.71 to 2.00 would be the coefficient of the expected reduction. This reduction overestimated the reported decrease in judged length only slightly. On the other extreme, where length distribution was most unequal, subjects might simply have operated on the longer line and added some, without really summing the two lengths on the subjective scale. These explanations are in accord with the two-stage expression and the data cf the present study.

The input exponent also allows for the direct recovery of the Munsell value relation among stimuli. The subjective scale differences for all stimulus pairs whose members are separated by equal Munsell scale vlaues were averaged, and the arithmetic means are given in Table 3. For example, the four stimulus pairs 1-5, 2-6, 3-7, and 4-8 are all separated by 3.00 Munsell steps, so the four subjective scale values (Ψ_{ij}) of the differences were averaged to yield the values 1.02 for lightness and 1.17 for darkness. The means were then multiplied by a constant (a' = 3.00 for lightness and a' = 2.59 for darkness), which did not change the essential ratio properties but simply brought them into line with the Munsell step values. Inspection of Table 3 reveals that the subjective scale derived from application of the two-stage model is virtually identical to the standard Munsell value scale. This finding further confirms the operation of $\underline{k}$ and, by implication, $\underline{m}$, as input and

Table 3

Comparison between Munsell scale interval and subjective value of interval for lightness and darkness.

Munsell Interval	Number of Pairs	$(\phi_j^k - \phi_i^k)$ Lightness	Mean Darkness	Lightness Meanx3.00	Darkness Meanx2.59
.75	7	.25	.29	.75	.75
1.50	6	.51	.58	1.53	1.50
2.25	5	.76	.88	2.28	2.27
3.00	4	1.02	1.17	3.06	3.03
3.75	3	1.27	1.46	3.81	3.78
4.50	2	1.52	1.75	4.56	4.53
5.25	1	1.77	2.04	5.31	5.28

output exponents of the power function. These findings also agree in principle with Curtis (1970), who attributed the nonlinearity of the relationship between category and magnitude estimation judgments to differences in the output stage transformation.

There has been considerable debate on the effects of stimulus range selected for an experiment on the exponent obtained. Poulton (1967) maintained that the (geometric) range of the stimuli is a major determining factor for the value of the exponent of the power function. He concluded that the exponent can be easily manipulated by the experimenter, who is free to select one of many possible series of stimuli. Teghtsoonian (1971) went even further, and in a somewhat different direction, by postualting a "common scale of sensory magnitude for a wide variety of perceptual continua." The experimenter's choice of stimuli is usually determined largely by the dynamic range for the continuum in question, and the subjects' response scale is usually constant at about 1.5 log units. The dynamic range, then, rather than the experimenter's arbitrary selection of stimuli, plays a major role in determining the value of the exponent. Stevens (1971) indicated that in order to meaningfully compare exponents across continua (or even studies for the same continuum), the experimenter must be certain that the subjective ranges are comparable.

It is possible that the two-stage model allows for

more precise statements concerning stimulus and subjective ranges than has previously been possible. One simply can assume a value for *k* established by previous research (such a value is fairly stable at about .35 for judgments of gray) and then construct a subjective scale based on values of ϕ^k. This would allow one to look at subjective range and, as well, spacing or location of a standard stimulus within the range.

Besides being able to compare these conditions across continua, application of the *k* value can allow for a selection of stimuli symmetrically representative of the subjective scale, regardless of the continuum under investigation. Stimuli are often selected according to what can be comfortably endured by the subjects or according to what seems to be representative to the experimenter. Both methods of selection are attempts to select a balanced set of stimuli from within the dynamic range, but there seems to be no precise way for doing such. The gray continuum, when measured in per cent reflectance under constant lighting, presents the unusual condition of being bounded beneath by black = 0.0 per cent and above by white = 100.0 per cent. It may be desirable to select maximum and minimum stimuli such that the distance between the minimum stimulus and black is equal to the distance between white and the maximum stimulus. This may be referred to as "balanced stimulus range", and the relationship

$$\phi_{max}^{k} + \phi_{min}^{k} = 100$$

must be satisfied by such a set of stimuli. Units are in per cent rather than proportion reflectance in order to simplify subjective scale units. For the present study, selection of a maximum stimulus whose physical value is 46.8% would require a minimum of 1.7%, which is smaller than the value of 3.1% actually selected. If $\underline{k}$ were equal to .39, as it has been computed under certain conditions (Curtis & Rule, 1972b), then the stimuli of 46.8% and 3.08% provide for a balanced stimulus range. One problem is that the selection of balanced stimuli will be quite sensitive to changes in $\underline{k}$, and $\underline{k}$ is determined empirically from judgments of stimuli themselves, under a particular set of experimental conditions. It is likely that an average of $\underline{k}$ taken from several studies will be more stable and less the result of systematic bias than would be a single determination of $\underline{k}$ for a continuum. Also, it is possible that an expression for dynamic range itself in terms of the input exponent would be worthwhile to explore. The hyperbolae of Figure 18 are for several possible values of $\underline{k}$. The physical values of the maximum and minimum stimuli can be read off the appropriate axes. Though the gray continuum, as mentioned, has the convenience of both upper and lower bounds, there is some evidence which may allow for the use of an estimated upper bound for other continua. Irwin and Corballis (1968) indicated that the softness function appeared to be affected by a high threshhold, just as the loudness function is affected by a low threshhold.

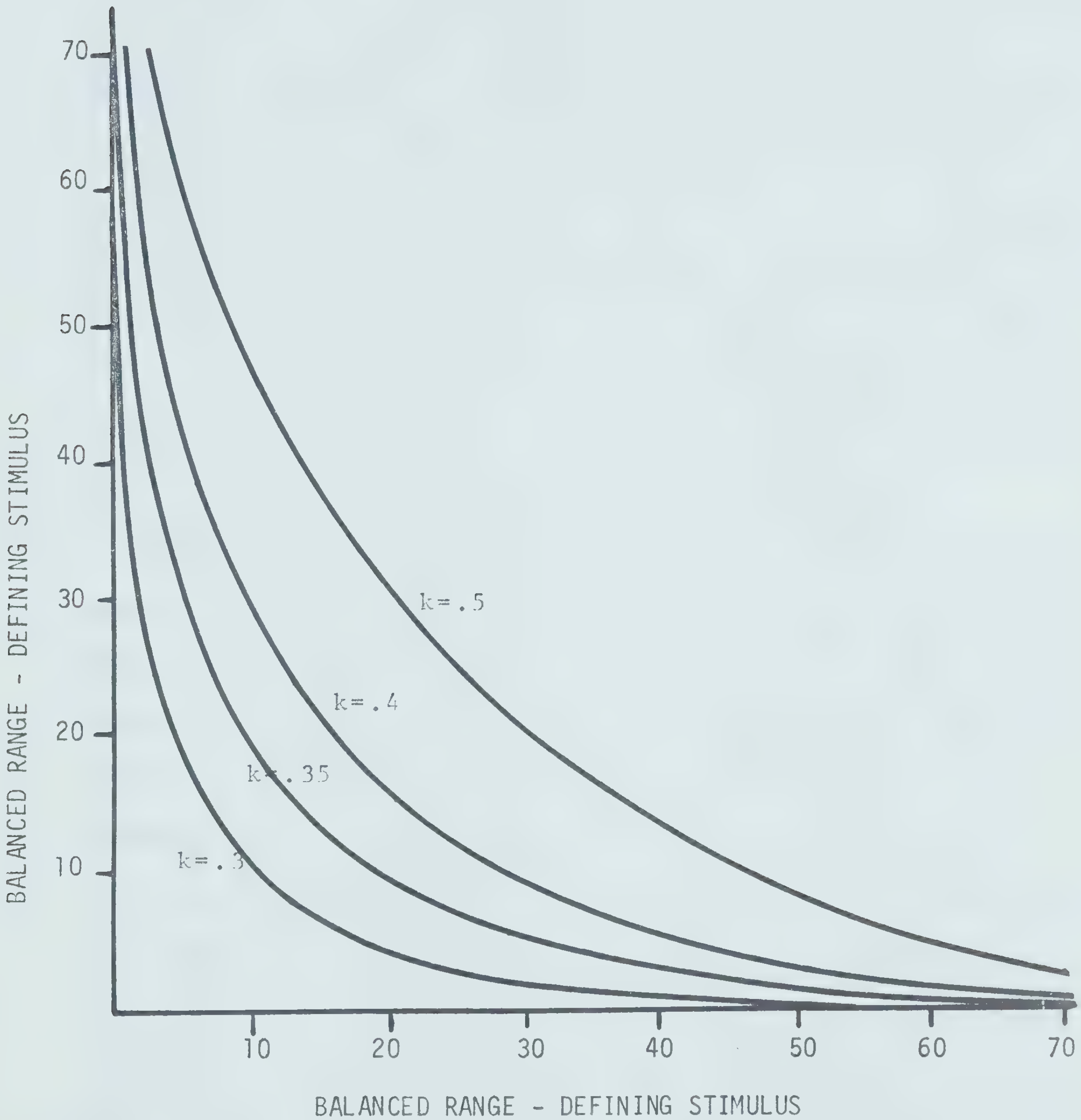

Figure 18. One range-defining stimulus (maximum or minimum in series) as a function of the other. Curves represent different values of the input exponent, _k_.

The value of such a high threshhold may be useful in estimating an effective upper bound.

Some data of Weiss (1972) can be examined in terms of the notion of balanced stimulus range. Weiss' Figure 5 shows the geometric means for "grayness" (i.e., darkness) judgments of ten stimuli plotted against reflectances on double logarithmic coordinates. The relation was described by Weiss as being severely nonlinear. The reflectance of the smallest stimulus was 3.1%, and the largest was 83%. The data for the smaller seven of the ten stimuli were almost perfectly linear on log-log coordinates in Weiss' plot; for the higher three stimuli the magnitude estimations decreased rapidly, giving the entire graph the appearance of being in conflict with the power function model for magnitude estimation. It should be noted, however, that for an input exponent anywhere less than .4, which it most surely is, the higher three stimuli, all greater than 50% reflectance, throw the series out of balance on the subjective scale (see Figure 18). The higher the unbalanced reflectance, the greater the deviation in the graph from linearity. An estimate of the slope of the line fitted through the first seven data points is .6, quite in line with the results of the present study.

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APPENDIX I

Values of the parameters for the $1/J^2$ weighting, in which squared deviations of J from values predicted by the functions are weighted inversely proportional to J^2, for both single stimulus and difference data.

$$a\phi^n + b$$

	a	n	b
Lightness	97.24	.57	-3.64
Darkness	7.01	-.55	-6.45

$$a(\phi_j^k - \phi_i^k)^m + b$$

	a	k	m	b
Lightness	215.12	.29	2.04	3.33
Darkness	216.86	.36	1.81	5.53

APPENDIX II

Internal consistnecy of corrected* difference judgments.

LIGHTNESS

Stimulus Pair	Direct Estimate	Indirect Estimate	Interior Pairs
1,8	6.36	6.45	1,2+2,8
		6.00	1,3+3,8
		6.24	1,4+4,8
		6.22	1,5+5,8
		6.33	1,6+6,8
		5.99	1,7+7,8
1,7	5.34	5.97	1,2+2,7
		5.38	1,3+3,7
		5.02	1,4+4,7
		5.51	1,5+5,7
		5.39	1,6+6,7
1,6	4.31	4.57	1,2+2,6
		4.06	1,3+3,6
		4.09	1,4+4,6
		4.70	1,5+5,6
1,5	3.59	4.03	1,2+2,5
		3.33	1,3+3,5
		3.36	1,4+4,5
1,4	2.27	2.94	1,2+2,4
		2.32	1,3+3,4
1,3	1.56	1.65	1,2+2,3
2,8	5.32	4.96	2,3+3,8
		5.78	2,4+4,8
		5.53	2,5+5,8
		5.46	2,6+6,8
		5.49	2,7+7,8
2,7	4.84	4.34	2,3+3,7
		4.56	2,4+4,7
		4.82	2,5+5,7
		4.52	2,6+6,7
2,6	3.44	3.02	2,3+3,6
		3.63	2,4+4,6
		4.01	2,5+5,6
2,5	2.90	2.29	2,3+3,5
		2.90	2,4+4,5
2,4	1.81	1.28	2,3+3,4

*Correction is for additive response constant and nonlinearity of output function.

Continued on next page.

Stimulus Pair	Direct Estimate	Indirect Estimate	Interior Pairs
3,8	4.44	4.73	3,4+4,8
		4.40	3,5+5,8
		4.52	3,6+6,8
		4.49	3,7+7,8
3,7	3.82	3.51	3,4+4,7
		3.69	3,5+5,7
		3.58	3,6+6,7
3,6	2.50	2.58	3,4+4,6
		2.88	3,5+5,6
3,5	1.77	1.85	3,4+4,5
4,8	3.97	3.72	4,5+5,8
		3.84	4,6+6,8
		3.40	4,7+7,8
4,7	2.75	3.01	4,5+5,7
		2.90	4,6+6,7
4,6	1.82	2.20	4,5+5,6
5,8	2.63	3.13	5,6+6,8
		2.57	5,7+7,8
5,7	1.92	2.19	5,6+6,7
6,8	2.02	1.73	6,7+7,8

DARKNESS

1,8	9.88	10.12	1,2+2,8
		9.33	1,3+3,8
		10.58	1,4+4,8
		10.29	1,5+5,8
		9.12	1,6+6,8
		9.30	1,7+7,8
1,7	8.57	9.19	1,2+2,7
		8.27	1,3+3,7
		8.28	1,4+4,7
		8.93	1,5+5,7
		7.74	1,6+6,7
1,6	6.52	7.12	1,2+2,6
		6.40	1,3+3,6
		7.55	1,4+4,6
		7.63	1,5+5,6
1,5	5.86	5.66	1,2+2,5
		5.22	1,3+3,5
		6.37	1,4+4,5

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Stimulus Pair	Direct Estimate	Indirect Estimate	Interior Pairs
1,4	4.33	4.07	1,2+2,4
		4.96	1,3+3,4
1,3	2.57	3.16	1,2+2,3
2,8	8.71	8.51	2,3+3,8
		8.91	2,4+4,8
		8.68	2,5+5,8
		8.31	2,6+6,8
		8.51	2,7+7,8
2,7	7.78	7.50	2,3+3,7
		6.61	2,4+4,7
		7.32	2,5+5,7
		6.93	2,6+6,7
2,6	5.71	5.58	2,3+3,6
		5.88	2,4+4,6
		6.02	2,5+5,6
2,5	4.25	4.40	2,3+3,5
		4.70	2,4+4,5
2,4	2.66	3.12	2,3+3,4
3,8	6.76	7.64	3,4+4,8
		7.08	3,5+5,8
		6.43	3,6+6,8
		6.48	3,7+7,8
3,7	5.75	5.34	3,4+4,7
		5.72	3,5+5,7
		5.05	3,6+6,7
3,6	3.83	4.61	3,4+4,6
		4.42	3,5+5,6
3,5	2.65	3.43	3,4+4,5
4,8	6.25	6.47	4,5+5,8
		5.82	4,6+6,8
		4.68	4,7+7,8
4,7	3.95	5.11	4,5+5,7
		3.44	4,6+6,6
4,6	3.22	3.81	4,5+5,6
5,8	4.43	4.37	5,6+6,8
		3.80	5,7+7,8
5,7	3.07	2.99	5,6+6,7
6,8	2.60	1.95	6,7+7,8

APPENDIX III

Geometric means of single stimulus data.

Stimulus*	Geometric mean of judgments of:	
	Lightness	Darkness
1	61.10	4.26
2	49.84	5.84
3	42.14	8.08
4	35.12	10.09
5	27.37	15.76
6	20.51	19.84
7	15.70	29.13
8	9.59	40.91

*Stimuli are listed in order of reflectance, with Stimulus 1 having the greatest reflectance.

APPENDIX IV

Geometric means of difference data.

Stimulus pair*	Mean of judgments of difference in: Lightness	Darkness
1,2	4.56	7.18
1,3	5.70	10.56
1,4	8.40	18.37
1,5	16.03	27.44
1,6	21.57	31.93
1,7	31.36	48.32
1,8	42.98	60.41
2,3	3.54	8.00
2,4	6.53	10.91
2,5	11.57	17.95
2,6	14.93	26.47
2,7	26.29	41.62
2,8	31.15	49.55
3,4	3.85	7.12
3,5	6.38	10.87
3,6	9.47	15.85
3,7	17.64	26.68
3,8	22.70	33.66
4,5	4.46	8.85
4,6	6.58	13.10
4,7	10.77	16.42
4,8	18.84	30.09
5,6	4.49	8.06
5,7	6.95	12.48
5,8	10.14	18.89
6,7	4.45	6.76
6,8	7.31	10.66
7,8	3.69	5.92

*Stimuli are numbered in order of reflectance, with stimulus 1 having the greatest reflectance.